An Accurate and Compact 3D Positioning System for a Moving Target by Integrating Extended Phase Accordance Method and Particle Filter

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***Abstract*—This paper describes a technique for accurate three-dimensional (3D) position estimations by integrating the extended phase accordance method (EPAM) and the particle filter (PF). Unlike existing positioning systems, EPAM can obtain not only the 3D position but also the instantaneous velocity of the target by using ultrasonic transducers installed with a short baseline. PF uses the position and velocity obtained by EPAM to compute more accurate estimations. The experimental results proved that the integration of EPAM and PF could improve the positioning accuracy. When a single compact ultrasonic receiver unit mounting four ultrasonic sensors to form a 76.2 mm square was used, the average 3D position error was improved from 40 mm to 29 mm. The 95th percentile of the error was also improved to 114 mm while that of EPAM without PF was 160 mm.**

***Index Terms*—Ultrasonic, Extended Phase Accordance Method, Particle Filter, Gestural Interface, Indoor Positioning System**

I. INTRODUCTION

Positioning techniques are essential to location-aware computing, which provides context-aware services to users. In outdoor areas, a global positioning system (GPS) is widely used to obtain location information. However, the performance of GPS deteriorates in indoor environments because buildings and walls block the GPS signal. Therefore, many research groups have attempted to develop indoor positioning systems by using such approaches as WiFi, visible light, infrared and ultrasonic communications [1]. Ultrasonic waves are a promising approach for indoor positioning because of their cost effectiveness. Cricket [2] and Active Bat [3] are examples of noteworthy studies on ultrasonic positioning systems.

One of the biggest problems in most existing ultrasonic 3D positioning systems is the tradeoff between their accuracy and size, which is related to the baseline between beacons. If the baseline is short, the accuracy of localization systems generally degrades. On the other hand, in order to obtain a high level of positioning accuracy, the beacons must be dispersed widely. Therefore, in most existing systems, an adequate level of positioning accuracy is ensured while increasing the installation cost unwillingly.

In this paper, we propose a positioning method with sufficient accuracy for indoor positioning by using only one compact receiver unit. To achieve low deployment costs with high positioning accuracy, we integrated the extended phase accordance method (EPAM) [4] and the particle filter (PF) [5]. When implementing a PF, it is common to model the time transitions of the variables to be filtered so that we can predict their values in the next time step. This modeling is important for implementing the PF so that filtered estimates do not diverge from true values. However, it is difficult to express the motion of a free-moving object with predefined motions such as linear or circular motions. In order to handle this problem, we use EPAM as a positioning method. The main advantage of EPAM is its ability to estimate the instantaneous velocity of a moving target as well as its accurate position. The velocity information is quite useful for predicting the position in the next time step.

The remainder of the paper is organized as follows. In Section II, we briefly describe EPAM and introduce the proposed particle filtering method. In Section III, we show experimental results that illustrate the performance of the integration of EPAM and PF. Section IV gives the conclusions of this paper and mentions future work.

II. TRACKING ALGORITHM

A. 3D position and velocity detection

We can calculate the 3D position of the target when EPAM can conduct distance measurements to a moving target from three or more microphones, as shown in Fig. 1. EPAM also allows us to obtain the velocity vector $\boldsymbol{V}$ of the target. The velocity $\boldsymbol{v}_i$ at the ith microphone, is the projection of $\boldsymbol{V}$ to the line connecting the target and the ith microphone. Thus, $\boldsymbol{V} \cdot \boldsymbol{u}_i = \boldsymbol{v}_i$ are satisfied. Here, $\boldsymbol{u}_i$ is a unit vector directed from the transmitter to the ith microphone. A 3D positioning system using EPAM has been proposed previously [6].

B. Motion models

In this subsection, a motion model of a moving target is described. In a discrete time setting, it is common to model motions using a Markov stochastic process. We represent the

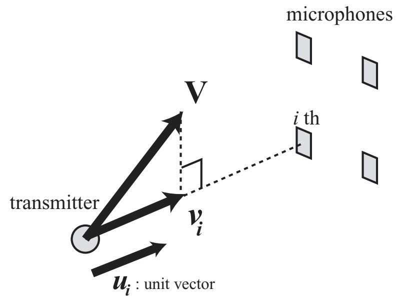

Fig. 1. Geometric relation between the transmitter and microphones.s

target state at time t as a 6×1 real vector $\boldsymbol{\mu}_t$:

$$\boldsymbol{\mu}_t = \begin{pmatrix} \boldsymbol{X}_t \\ \boldsymbol{V}_t \end{pmatrix}, \tag{1}$$

where the 3×1 vector $\boldsymbol{X}_t$ indicates the position of the target. Here, the state transition can be modeled as follows, assuming that the velocity of the moving target is constant:

$$\begin{aligned} \boldsymbol{\mu}_{t+1} &= \begin{bmatrix} \boldsymbol{I}_3 & T \cdot \boldsymbol{I}_3 \\ \boldsymbol{O}_3 & \boldsymbol{I}_3 \end{bmatrix} \cdot \boldsymbol{\mu}_t + \boldsymbol{\epsilon}_t \\ &= \boldsymbol{F} \cdot \boldsymbol{\mu}_t + \boldsymbol{\epsilon}_t \,, \end{aligned} \tag{2}$$

where $\boldsymbol{F}$ is a transition matrix that depends on the measurement interval T. $\boldsymbol{\epsilon}_t$ represents the error vector introduced by the state transition, including the effects of unknown accelerations. The measurement can be also described as:

$$\boldsymbol{z}_t = \boldsymbol{H} \cdot \boldsymbol{\mu}_t + \boldsymbol{u}_t \,, \tag{3}$$

where $\boldsymbol{z}_t$ is the measurement vector. Its components are the values detected by the measurement system. $\boldsymbol{u}_t$ is the measurement error. As previously mentioned, EPAM can obtain the 3D position and the velocity vector of the target. Thus, the measurement matrix $\boldsymbol{H}$ is a 6×6 identity vector and there are six z components in our implementation.

Many existing positioning systems can detect only the position of the target. For this reason, the following two filtering models have been widely used in most positioning systems.

- $P\ model$: A system has only position components as its state vector $\boldsymbol{\mu}_t$. In this model, the state transition is modeled assuming that the target is in the same position in the next time step.
- $PV\ model$: A system has both position and the velocity components. In this model, the measurement matrix is as follows, because the system cannot observe the instantaneous velocity.

$$\boldsymbol{H} = [\boldsymbol{I}_3 \ \boldsymbol{O}_3] \,. \tag{4}$$

In this paper, we compared the performance of these two types of filter models with that of the proposed system.

C. Particle filter

Particle filters (PFs) are used to estimate the state of a system as a statistical state in which the corresponding probability density function (PDF) is approximated numerically. A PF uses a set of N_p particles, each of which has a candidate state vector, and a corresponding weight. In order to approximate the state posterior PDF $p(\boldsymbol{\mu}_t|\boldsymbol{z}_t)$, a PF uses a set of particles as samples extracted from the posterior distribution, instead of representing them in a parametric form. With this representation, the PDFs of $\boldsymbol{\epsilon}_t$ and $\boldsymbol{u}_t$ are not restricted to be Gaussian. The 3D position calculation process in our implementation is nonlinear. If the distances are measured with Gaussian noise, the 3D positions are estimated with non-Gaussian noise. Fig. 2 expresses this nonlinearity. We generated 10,000 sets of distance measurements with Gaussian noise and calculated 3D positions with those distance data. We can confirm that the noise on the 3D position estimates is non-Gaussian. By using the PF, we can handle this noise distribution.

We now briefly describe the algorithm of the proposed PF. The operations 2–4 are performed iteratively.

1) Initialization: The posterior (approximated) particles at time t are denoted as $\{\boldsymbol{\mu}^+_{t,i}\}_{i=1}^{N_p}$ and the corresponding set of weights as $\{w_{t,i}\}_{i=1}^{N_p}$. In our implementation, each initial particle is generated by sampling from a mixture of $\boldsymbol{z}_1$ and zero-mean Gaussian noise with an appropriate variance.

2) Prediction: In the prediction phase at time t, new *a priori* particles $\{\boldsymbol{\mu}^-_{t,i}\}_{i=1}^{N_p}$ are obtained. The prediction process applies the state propagation equation (2) to the posterior particles $\{\boldsymbol{\mu}^+_{t-1,i}\}_{i=1}^{N_p}$. The transition noise $\boldsymbol{\epsilon}_t$ is set to be a zero-mean Gaussian distribution and does not depend on the time t ($\boldsymbol{\epsilon}_t = \boldsymbol{\epsilon}$):

$$\boldsymbol{\mu}^-_{t,i} \sim \mathrm{N}\left(\boldsymbol{F} \cdot \boldsymbol{\mu}^+_{t-1,i}, \, \boldsymbol{\epsilon}\boldsymbol{\epsilon}^T\right), \, i \in \{1, \ldots, N_p\}. \tag{5}$$

3) Weight evaluation: The weight of each particle is used to incorporate the measurement $\boldsymbol{z}_t$ into the particle set. Thus, the weight is the probability of the measurement $\boldsymbol{z}_t$ under the particle $\boldsymbol{\mu}^-_{t,i}$, which is given by:

$$w_{t,1} = p(\boldsymbol{z}_t|\boldsymbol{\mu}^-_{t,i}). \tag{6}$$

In order to decide the likelihood function $p(\boldsymbol{z}_t|\boldsymbol{\mu}^-_{t,i})$, we analyze the noise distributions numerically as shown in Fig. 2. Thus, the non-Gaussian noise distribution is taken into account in the weight evaluation process.

4) Resampling: In this phase, the particles are resampled based on their weights so as to obtain the posterior particles $\{\boldsymbol{\mu}^+_{t,i}\}_{i=1}^{N_p}$. The previous particles $\{\boldsymbol{\mu}^+_{t-1,i}\}_{i=1}^{N_p}$ are replaced with the newly extracted particles. Through this process, the particles with lower importance weights are discarded.

III. Experiment

Experiments to evaluate the performance of the proposed filtering method were conducted. 3D positions of a user's hand moving freely were estimated as shown in Fig. 3. An ultrasonic transmitter (T40-16; Nippon Ceramic Corporation, Japan) was mounted on the user's hand. A receiver unit

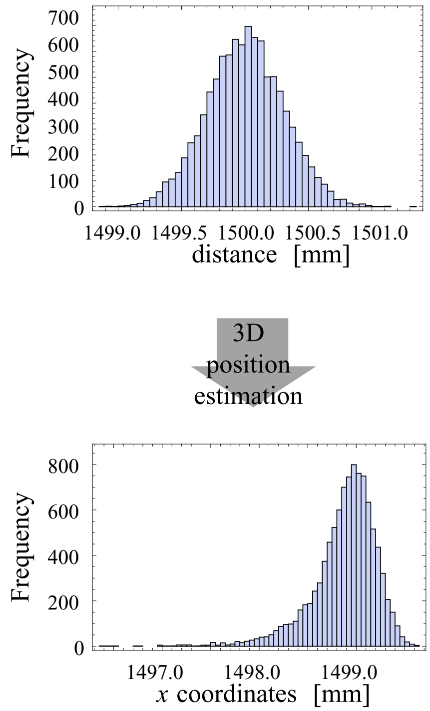

Fig. 2. 3D estimates are obtained with non-Gaussian noise. Upper: distance data with Gaussian noise; lower: 3D position estimates calculated with the above distance data.

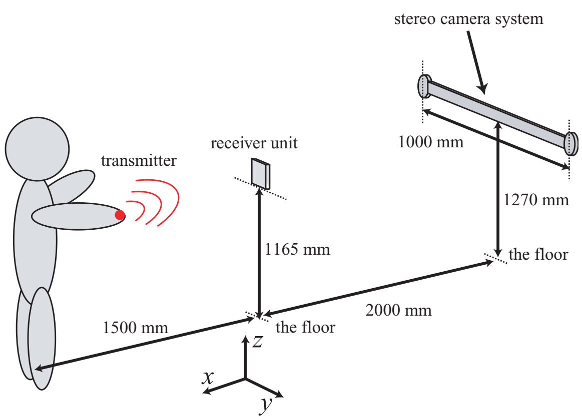

Fig. 3. Experimental setup. All receiver units and cameras are on the same plane.

included four ultrasonic receiver microphones (SPM0404UD5; Knowles Acoustics, USA), which were arranged to form a 76.2 mm square. Fig. 4 shows the transmitter and the receiver unit, respectively. To evaluate the positioning accuracy, a stereo camera positioning system was used to acquire the 3D position of the transmitter augmented with three IR-LEDs for reference. The positioning error of this camera system was at most 3 mm. The system could capture the moving ultrasonic transmitter at about 10 frames per second. In the experiment, 72 position datasets were obtained.

Three types of PFs (P model, PV model and proposed PF) were applied to the obtained data. The number of particles used N_p was 300. We evaluated the positioning accuracy by

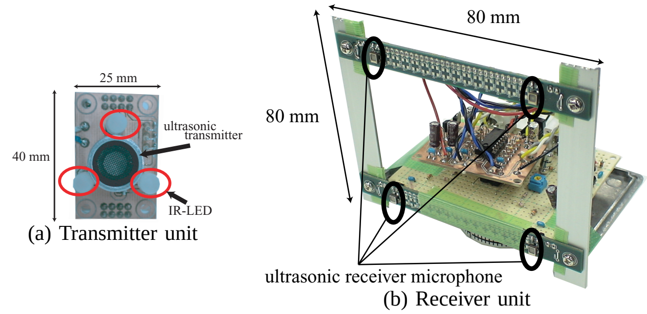

Fig. 4. System components.

TABLE I
STANDARD DEVIATIONS OF 3D POSITIONING ERROR ESTIMATES ABOUT EACH AXIS (x, y, z) AND ERROR DISTANCE.

	Positioning error about each axis [mm]	Error distance [mm]
Observed	(38.5971, 70.5439, 45.1198)	39.9038
Filtered	(24.2199, 56.2565, 33.2028)	28.8900

errors about each axis and Euclidian distance errors between the estimated and true positions. Table I shows the standard deviations of the positioning errors. The results show that all the values were improved by the proposed filter. For the standard deviation of the error distance, 27.6% improvement was confirmed. Fig. 5 shows the error cumulative distribution functions (CDFs) of the positioning estimates provided by the raw measurements and three types of PF. The figures show the improvement in the errors. For example, by using the proposed PF, the probability that the error of one measurement was less than 100 mm was improved from 72% to 90%.

IV. Conclusion and Future Work

We have described an accurate and compact system for 3D position estimates that integrates EPAM and PF. The experimental results proved that this integration effectively improved the tracking accuracy of a moving target. The proposed system achieved 29 mm standard deviations. Moreover, the error CDF was also improved. The probability that the position measurement had an error less than 100 mm was improved from 72% to 90%. These improvements were achieved because the velocity information measured with EPAM was useful for properly predicting the state representing the position and velocity of the target in the next time step.

We are now conducting investigations on the phase characteristics of ultrasonic receiver microphones, which produce distance measurement errors depending on incident angles of ultrasonic waves. We have already confirmed that these characteristics are reproducible and the distance measurement errors can be regarded as systematic errors. Thus, by measuring the microphone's characteristics, we can obtain a compensation function that reduces estimation errors. We expect to improve the positioning accuracy further, by implementing the compensation function in the PF.

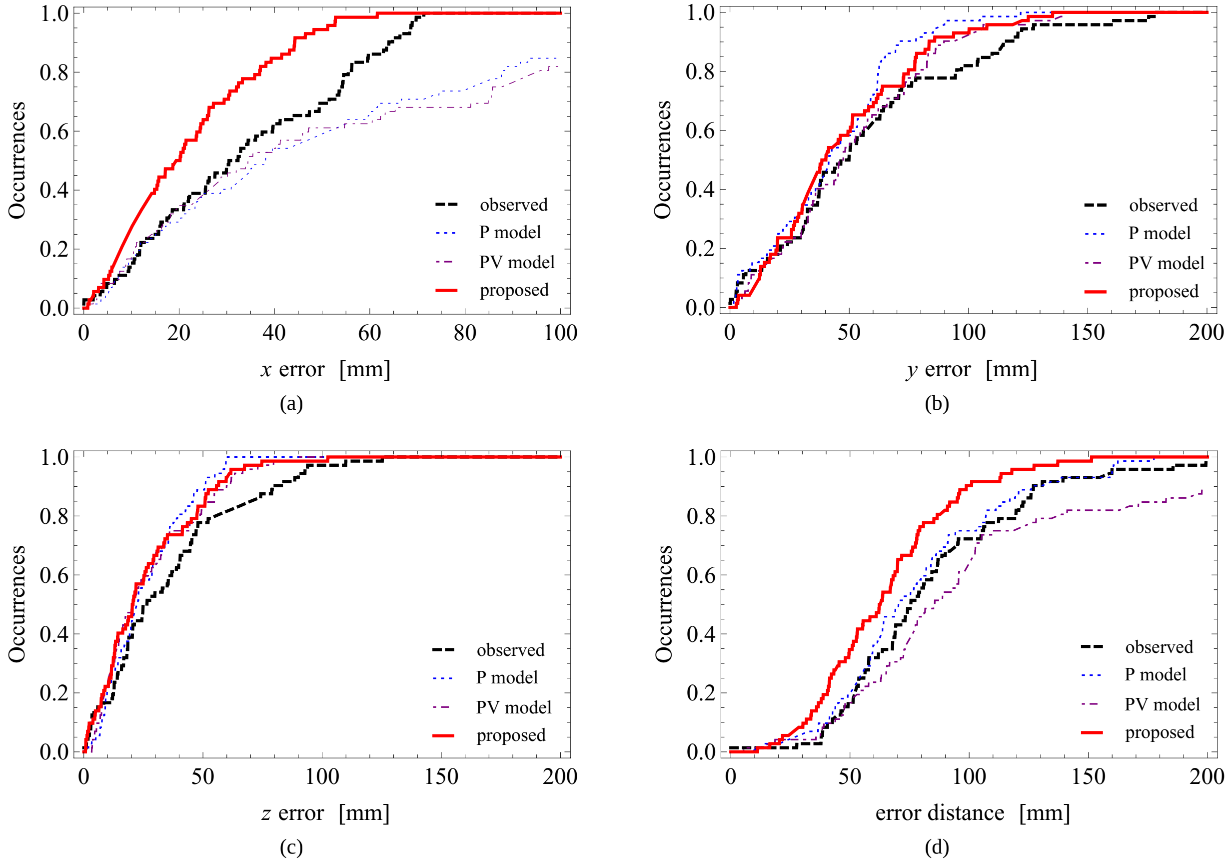

Fig. 5. Error CDFs of the different PFs: (a) x-axis error; (b) y-axis error; (c) z-axis error; (d) 3D position error.

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