

# Utility Based Node Selection Scheme for Cooperative Localization

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**Abstract**—Cooperative positioning is a popular approach for indoor localization, which significantly outperforms conventional positioning techniques. This paper presents a method for iterative node localization, with a relatively small number of initial anchor nodes. In order to reduce error propagation in iterative schemes, it is necessary to use only reliable information among nodes. We introduce a reference node selection strategy based on utility functions. The method is completely distributed and involves only information from local neighborhood. Compared to the nearest neighbor and random node selection scheme, simulation results show that selecting subsets of nodes with highest utility values leads to more accurate position estimates.

**Keywords**—multilateration; utility; node selection

## I. INTRODUCTION

Localization techniques rely on internode measurements and distance estimates to fixed anchor nodes with known coordinates. Cooperation between nodes is used in cases when conventional positioning techniques do not perform well due to lack of existing infrastructure or obstructed indoor environment. Both cooperative and conventional techniques usually consist of two stages: 1) a ranging phase where nodes estimate distances to their neighbors by measuring some distance dependent signal metric, and 2) an algorithm where the ranging information is used for position calculation.

One simple way for position calculation is linear least squares lateration. In the ideal case, the coordinates of the unknown node would correspond to the point of intersection of at least three circles with center in anchor node's coordinates and radius equal to distance to each reference node. Due to erroneous distance estimates these circles do not intersect in one single point, and least squares optimization is applied to minimize the sum of squared residuals. Hence it leads to a nonlinear optimization problem which requires appropriate initial estimates and is considered too expensive [1]. An alternative approach is to use linearized expressions and calculate the position estimate by means of linear least squares (LLS) approach. It is not an optimal estimator but yields a low complexity solution with reasonable accuracy [1]. Least squares approach has been adopted in cooperative positioning schemes in [2]-[6]. While all these algorithms are deterministic, i.e., their aim is to find the deterministic location, statistical methods such as belief propagation [7] and factor graphs [8] aim to estimate the maximum a posteriori location using a set of observations and a priori probability distributions of

node locations. Several centralized solutions have been proposed such as multidimensional scaling [9] and convex optimization [10].

In particular, we will consider a distributed localization approach, namely iterative multilateration. Once an unknown node estimates its position, it becomes an anchor and broadcasts its position estimate to all neighboring nodes. The process is repeated until all nodes that can have three or more reference nodes obtain a position estimate. It only involves information within local neighborhood and hence reduces communication cost. However, it suffers from error propagation. As a newly localized node is becoming a new anchor for its neighbors, the estimation error of the first node can propagate to other nodes and eventually get amplified. Excessive iterations could lead to widespread error distribution throughout the network, leading to abundant error in large topologies. The effect may also arise in global methods such as MDS or SDP, but the global constraints are likely to balance against each other and hence make global methods less vulnerable. Hence it is important to choose reference nodes carefully and hereby reduce error accumulation by taking into account uncertainties in reference nodes estimates.

The rest of the paper is organized as follows. Section II gives an overview on factors that contribute to localization error. In Section III we describe our node selection method based on utility functions, and in Section IV we present simulation results to depict the performance of our proposed method. Conclusions are given in Section V.

## II. LOCALIZATION ERROR

Localization error is a function of several factors, such as number of anchor nodes, node density, network topology etc. In addition to noisy distance estimates and reference node geometry, the error propagation problem is also resulting from use of erroneous estimates as virtual anchors in subsequent iterations. An unknown node receives information from many neighbors, some of which are virtual anchors with a degree of uncertainty in their estimates. Therefore not all of those links have the same level of "usefulness", even if localization accuracy increases with the number of used reference nodes, from the information theory perspective. Especially the geometry of used reference nodes has been shown to have a high impact on lateration.

The geometric conditioning on localization accuracy is derived in the GDOP (geometric dilution of precision) metric [11]. A commonly used tool to

describe the error bounds on location estimates is the Cramer Rao Lower Bound, which will be explained in detail in II.B. In [12] it was shown that the effect of inexact location knowledge of reference nodes on error bounds is equivalent to the increase of variance of RSS-based distance estimation.

#### A. RSS based distance estimation

In case of received signal strength, ranges are first estimated and then LS techniques are applied on these ranges. We use the standard lognormal model for RSS with path loss parameter  $n_p$  and shadowing variance  $\sigma_{RSS}^2$ . Assuming that the received power  $P_{ij}$  between nodes  $i$  and  $j$  is lognormal, the random variable  $P_{ij}$  (dBm) =  $10\log P_{ij}$  is Gaussian and the maximum likelihood distance estimate with bias correction is given by [13]:

$$d_{i,j} = d_0 10^{\frac{P_0 - P_{i,j}}{10n_p}} e^{\frac{\gamma^2}{2}}, \quad (1)$$

where  $\gamma = \frac{\sigma_{RSS} \ln 10}{10n_p}$  and  $e^{\frac{\gamma^2}{2}}$  is a multiplicative bias correction factor.

#### B. Cramer Rao Lower Bound

The Cramer Rao Lower Bound provides a lower bound on covariance of any unbiased estimator. It is calculated as the inverse Fisher Information Matrix (FIM). The Fisher information is a way of measuring the amount of information that an observable random variable carries about an unknown parameter upon which the likelihood function depends. Considering the log-likelihood function of random measurements  $f_{RSS}$  the FIM is given by

$$FIM = E\left\{\nabla f_{RSS} \cdot \nabla f_{RSS}^T\right\}, \quad (2)$$

where  $\nabla f_{RSS}$  is the gradient of the log-likelihood function. It can be shown that the CRLB for RSS based distance estimates for multilateration with  $N$  anchor nodes will be of the form [13]:

$$CRLB_{RSS} = \frac{1}{b} \frac{\sum_{i=2}^N d_{1,j}^{-2}}{\sum_{i=2}^{N-1} \sum_{j=i+1}^N \left(\frac{d_{1,i,j} d_{i,j}}{d_{1,i}^2 d_{1,j}^2}\right)^2}, \quad (3)$$

$$\text{where } b = \left(\frac{10n_p}{\sigma_{RSS} \ln 10}\right)^2.$$

CRLB captures information about node geometry and channel conditions (ranging quality). For diverse topologies the error bounds will be different. Since the variance of position estimates is associated to the mean error, the lower bound on variance can be seen as the upper bound on accuracy.

### III. NODE SELECTION

As concluded in the previous section, the output of a multilateration procedure is varying if different sets of references are chosen. Here we will focus on determining which combination of references results in best performance, in case when alternative reference nodes are available. In order to find a proper model that considers all the factors contributing to localization error, we will adopt some concepts from game theory and utility functions to the node selection problem.

Let us first review some concepts from game theory and see how these concepts can be adapted to the localization problem.

#### A. Utility function

A coalitional game consists of a set of players, an action set (strategy) for each player and a utility (payoff) for each player, measuring its level of satisfaction by assigning a value to a coalition. The players assess the usefulness of their strategies using their utility functions. In [14] a generic approach for coalition formation has been proposed, that has been applied to wireless communications, mostly in resource allocating and cognitive radio.

A cooperative game is the pair  $(N, v)$  where  $N$  is a finite set and  $v$  is its utility function. The elements of  $N$  are the players and any non-empty subset  $C \subseteq N$  is a coalition. In particular,  $N$  is called the grand coalition.

The main challenge here is to choose the appropriate utility function, i.e., how a node values different levels of performance. We can formulate the node selection optimization as the one that maximizes the accuracy subject to constraints given by nodes' limited processing capacity. The following parameters are relevant for reference node selection: number of references, their uncertainty (in case of virtual anchors), quality of range estimates and geometry. We assume that in each coalition exists a data fusion center which acts as coalition head [15]. We choose the closest anchor node, which is also used as reference for linearization [1] to serve as coalition head. Cooperation involves some cost per each anchor node  $c_i$ , associated with distance to the data fusion center  $d_{i,f_c}$ , but also yields a benefit  $B(C)$  in terms of improved accuracy when using a particular subset of nodes.

Assuming that the communication range is  $R$ , we define the utility function as:

$$v(C) = \begin{cases} 0, & \text{if } |C| < 3 \\ \frac{1}{crlb_{i \in C}} - \sum_{i \in C} \frac{d_{i,f_c}}{R}, & \text{otherwise} \end{cases}, \quad (4)$$

where the first term is the benefit indicator, while the second term represents the cost function related to the energy consumption required for communication. Since the true position of the unknown nodes is not accessible, the CRLB will be calculated using the estimated positions.

### B. Selection procedure

The simplest method is to perform an exhaustive search that evaluates the coalition value (4) for all possible sets of size  $\text{card}(C)$ , and then choose the set with the largest coalition value. This method is guaranteed optimal but the search time is exponential and the number of combinations is very large. It may be quite acceptable for small  $N$ . Pruned search methods were proposed in [16] to reduce the number of computations. Considering a low density scenario, and thereby a rather small number of candidate nodes, we will illustrate the algorithm performance using exhaustive search.

## IV. SIMULATION RESULTS

Let us assume a network consisting of  $N$  nodes,  $M$  of which are initially anchor nodes, and  $K = N - M$  remaining ones are unknown nodes. For evaluation we will consider one snapshot in the iterative algorithm, where a node analyzes its local neighborhood, and exchanges information with  $N_a$  reference nodes within communication range  $R = 30$  m. Among the candidate references the goal is to choose three of them,  $N_b = 3$ , which provide best performance. In our simulations, we assume that a node has 10 available candidate nodes, randomly placed within communication range of the unknown node, and 5 of them are assumed to be virtual anchors. Position is calculated using the LLS algorithm, using erroneous distance estimates as in (1). Ranging error is modeled using channel parameters  $n_p = 2.3$  and  $\sigma_{RSS}^2 = 3.92$  dB, as in [13].

In this work we assume a random, independent ranging error and all links have same channel conditions. We model the uncertainty of virtual anchors by associating a variance  $\omega^2$  to the true position. The number of possible combinations is:

$$\frac{N_a!}{N_b!(N_a - N_b)!} \quad (5)$$

Since the network density is not high, and having in mind limited communication range,  $N_a$  is relatively small and this approach is applicable. Otherwise we would have to switch to the pruned search methods. Each combination represents a possible coalition, and for all of them the coalition value is calculated based on (4). The subset of nodes with the highest value for utility function is used to estimate the position. Note that the set containing all  $N_a$  available reference nodes represents the grand coalition (the coalition of all nodes). However, since in our case there is cost associated with coalition formation, and we limit the selection to 3 nodes, the grand coalition will not form.

We take into account the virtual anchor uncertainty by means of variance  $\omega^2 = 1\text{m}$ , and adopt the results from [12], according to which imperfect location knowledge corresponds to the increase of variance, in case of RSS-based distance estimates.

Figure 1 shows through a scatter plot that higher coalition values lead to more accurate position estimates.

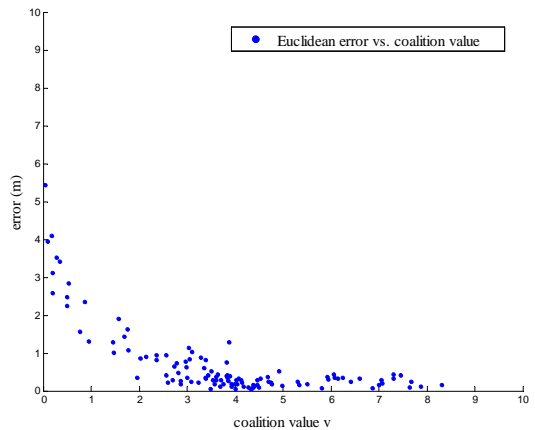


Figure 1: Simulation results

In Figure 2 we compare the error cumulative distribution functions of our proposed selection strategy against selection based on closest distances, as well as purely random reference selection. We perform 1000 independent runs. The 90<sup>th</sup> percentile for utility based selection is 3.5 m, which is an improvement of 39%, compared to 90<sup>th</sup> percentile of 5.8 m for closest distance, and 51% improvement with respect to the random case.

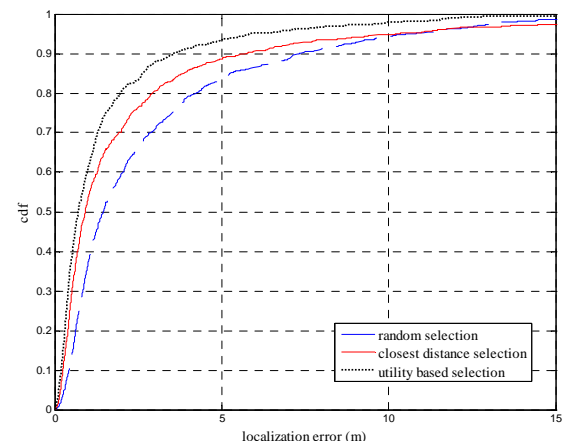


Figure 2: Error cdf

## V. CONCLUSION

In this paper we propose a technique for reference node selection with the objective to improve position accuracy in iterative multilateration based algorithms. Utility functions incorporate all information relevant for the node selection problem with respect to accuracy. The method is viable for distributed algorithms since it only involves information from the local neighborhood. For the considered scenario exhaustive search methods are still applicable; however for a dense deployment it would require too much computation. Therefore our future work will focus on more efficient search methods.

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