Predicting the expected accuracy for fingerprinting based WiFi localisation systems

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***Abstract*—WiFi localisation has become very popular in recent years. Most widely used are fingerprinting based techniques where a map of received signal strengths is used to infer the position based on comparing the current signal strength measurement to this map.**

Most research focuses on this inference itself and on the creation of accurate fingerprinting maps. In this paper we will not address those issues in depth but focus on analysing the fingerprint maps themselves in more detail. By looking closely into the maximum likelihood estimation for the position we will derive its expected uncertainty and show that it can be calculated for every possible position in advance from the fingerprint maps alone. This allows to derive an expected localisation accuracy map of the environment that can be used to assess and optimise the WiFi design based on localisation accuracy needs rather than relying on given access point placements solely based on coverage or signal-to-noise criteria.

I. Introduction

WiFi localisation has become very popular in recent years not only due to its ability to cover indoor areas but also due to the fact that it initialises faster and is much more energy efficient than GPS therefore not draining the batteries of modern smartphones so quickly.

A recent overview of the state of the art of indoor localisation has been given by [1]. Although alternative approaches exist, for instance based on machine learning techniques (cf. [2], [3], [4]) or decision trees (cf. [5]), fingerprinting based methods, i.e. using a pre-measured map of received signal strengths to infer the actual position based on such a measurement, are among the most popular (e.g. [6], [7], [8], [9], [10]).

The focus of this paper is not to propose yet another such algorithm but to analyse the localisation uncertainty structure inherently contained within the fingerprint maps. The achievable localisation accuracy has already been studied by [11], however they did not elaborate on the analysis of the fingerprints themselves. An empirical study into the accuracy of fingerprinting based localisation systems has been carried out by [12], who claim that there are fundamental limitations to what is achievable based on empirical evidence.

We will also not focus on the generation of the fingerprint maps, like for instance [13] or [14] who use Gaussian processes for this task, but assume that sufficiently dense fingerprints are given either obtained through extensive site surveying or through accurate prediction models. Like most of the fingerprinting based approaches we will use a Gaussian assumption for the distribution of signal strength values in a given position (eg. [15], [16]) for our analysis, although other distributions have been proposed based on empirical evidence (cf. [17]).

We will show that the expected uncertainty of the maximum likelihood estimate of the position based on a signal strength measurement can be derived from the gradient fingerprint map alone which is efficiently computable from it for every position in advance. This allows to generate an expected localisation accuracy map of the environment that can be used to assess and optimise the WiFi design based on localisation accuracy needs rather than relying on given access point placements solely based on coverage or signal-to-noise criteria (cf. [18]). We will compare this localisation accuracy map with the coverage map and the signal-to-noise ratio map for an example environment and show that the expected localisation accuracy behaves rather different to those other two measures suggesting that access point layouts should be based on the expected localisation accuracy as well if a WiFi based localisation system is to be installed.

II. Fingerprint Gradients and Smoothing

We will start by explaining how we obtain a smooth, grid-like fingerprint image and its corresponding gradient fingerprint image from a set of samples of the received signal strengths. Therefore we will assume that we are able to measure for every 2D position $\boldsymbol{x}$ a set of signal strengths

$$\boldsymbol{s} = \begin{pmatrix} s_1 & \cdots & s_K \end{pmatrix}^\mathsf{T} \tag{1}$$

from K different access points. Now the task of WiFi based localisation is to enable the deduction of the position $\boldsymbol{x}$ from such a signal strength measurement. For the sake of simplifying the notation we assume without loss of generality that the number of access points K is fixed and that a signal strength is available from each of the K access points at every position. Note though that in the following this assumption only needs to hold in small neighbourhoods of each position so that the results are easily transferable.

The first step in all fingerprinting based localisation systems is to create a signal strength map of the environment. We will assume that a set of signal strengths $\boldsymbol{s}^{(i)}$ has been measured corresponding to a set of locations $\boldsymbol{x}^{(i)}$ for this purpose. In order to obtain a smooth fingerprint grid from those discrete and unevenly distributed sampling points we use a Gaussian smoothing kernel of a given size σ and derive a 2D fingerprint map for each of the K channels separately as follows

$$F_k[\boldsymbol{x}] = \frac{\sum_{i\in\mathcal{N}[\boldsymbol{x}]} \exp \frac{-(\boldsymbol{x}^{(i)}-\boldsymbol{x})^\top(\boldsymbol{x}^{(i)}-\boldsymbol{x})}{2\sigma^2} s_k^{(i)}}{\sum_{i\in\mathcal{N}[\boldsymbol{x}]} \exp \frac{-(\boldsymbol{x}^{(i)}-\boldsymbol{x})^\top(\boldsymbol{x}^{(i)}-\boldsymbol{x})}{2\sigma^2}} \tag{2}$$

We consider only neighbouring points

$$\mathcal{N}[\boldsymbol{x}] = \{i \quad | \quad ||\boldsymbol{x}^{(i)} - \boldsymbol{x}|| < 4\sigma\} \tag{3}$$

in the sums because the exponentials are very close to zero outside this area anyway. This can be implemented very efficiently for large sample sets using spatial data structures such as k-d-trees or Quadtrees (cf. [19]).

The result of this operation is a smooth K channel discrete fingerprint image

$$\boldsymbol{F}[\boldsymbol{x}] = \begin{pmatrix} F_1[\boldsymbol{x}] & \cdots & F_K[\boldsymbol{x}] \end{pmatrix}^\top \tag{4}$$

In the following we will need not only the fingerprint image itself but also its gradient. This can be efficiently computed from the fingerprint map using standard image processing techniques (cf. [20]): for small values of h that are selected so that they match the resolution of the fingerprint image the gradient fingerprint image is given by

$$\nabla\boldsymbol{F}[\boldsymbol{x}] \approx \begin{pmatrix} \frac{\boldsymbol{F}\begin{bmatrix} x_1+h \\ x_2 \end{bmatrix} - \boldsymbol{F}[\boldsymbol{x}]}{h} & \frac{\boldsymbol{F}\begin{bmatrix} x_1 \\ x_2+h \end{bmatrix} - \boldsymbol{F}[\boldsymbol{x}]}{h} \end{pmatrix} \tag{5}$$

Note that equality holds in the limit $h \to 0$ and that therefore the smoothing kernel size σ must be large enough to allow a stable computation.

In the following section we will show how this gradient fingerprint image can be used to efficiently derive the expected localisation accuracy for every position.

III. Predicting the Localisation Accuracy

To derive the expected localisation accuracy we will assume the received signal strength measurement $\boldsymbol{s}$ at every position $\boldsymbol{x}$ to be Gaussian distributed around the fingerprint value $\boldsymbol{F}[\boldsymbol{x}]$ with a measurement covariance matrix given by $\boldsymbol{C}_{ss}$. Then its likelihood probability density function is given by

$$p\{\boldsymbol{s}|\boldsymbol{x}\} = \frac{\exp\left(-\frac{1}{2}(\boldsymbol{s}-\boldsymbol{F}[\boldsymbol{x}])^\top \boldsymbol{C}_{ss}^{-1}(\boldsymbol{s}-\boldsymbol{F}[\boldsymbol{x}])\right)}{\sqrt{(2\pi)^K \det \boldsymbol{C}_{ss}}} \tag{6}$$

The task of every signal strength based localisation algorithm is finding the position $\boldsymbol{x}$ with the highest probability given a specific signal strength measurement $\boldsymbol{s}$. Noting that the solution to a maximisation problem is equal to the solution of the minimisation of its negative logarithm, i.e.

$$\mathrm{argmax}_{\boldsymbol{x}} p\{\boldsymbol{x}|\boldsymbol{s}\} = \mathrm{argmin}_{\boldsymbol{x}} \left(-\log p\{\boldsymbol{x}|\boldsymbol{s}\}\right) \tag{7}$$

and furthermore assuming that every position is equally likely the position $\boldsymbol{x}$ of highest probability given a specific signal strength measurement $\boldsymbol{s}$ is found by minimising

$$\begin{aligned} -\log p\{\boldsymbol{x}|\boldsymbol{s}\} &= -\log \frac{p\{\boldsymbol{s}|\boldsymbol{x}\}p\{\boldsymbol{x}\}}{p\{\boldsymbol{s}\}} \\ &= \frac{1}{2}(\boldsymbol{s}-\boldsymbol{F}[\boldsymbol{x}])^\top \boldsymbol{C}_{ss}^{-1}(\boldsymbol{s}-\boldsymbol{F}[\boldsymbol{x}]) + Q \end{aligned} \tag{8}$$

subsuming all constant offsets into Q, which can be removed from the optimisation. Now making use of the gradient fingerprint image $\nabla\boldsymbol{F}[\boldsymbol{x}]$ described in the previous section and conceptually performing a Taylor series expansion of the fingerprint map at $\boldsymbol{x}_0$ as follows

$$\boldsymbol{F}[\boldsymbol{x}] = \boldsymbol{F}[\boldsymbol{x}_0 + \Delta\boldsymbol{x}] \approx \boldsymbol{F}[\boldsymbol{x}_0] + \nabla\boldsymbol{F}[\boldsymbol{x}_0]\Delta\boldsymbol{x} \tag{9}$$

this minimisation can be phrased as finding the displacement $\Delta\boldsymbol{x}$ minimising

$$\begin{aligned} \Omega[\Delta\boldsymbol{x}] = \frac{1}{2}&(\boldsymbol{s}-\boldsymbol{F}[\boldsymbol{x}_0]-\nabla\boldsymbol{F}[\boldsymbol{x}_0]\Delta\boldsymbol{x})^\top \boldsymbol{C}_{ss}^{-1} \\ &(\boldsymbol{s}-\boldsymbol{F}[\boldsymbol{x}_0]-\nabla\boldsymbol{F}[\boldsymbol{x}_0]\Delta\boldsymbol{x}) \end{aligned} \tag{10}$$

To find the minimum we calculate the derivatives with respect to $\Delta\boldsymbol{x}$ as follows

$$\begin{aligned} \frac{\partial\Omega[\Delta\boldsymbol{x}]}{\partial\Delta\boldsymbol{x}} &= \nabla\boldsymbol{F}[\boldsymbol{x}_0]^\top \boldsymbol{C}_{ss}^{-1}\nabla\boldsymbol{F}[\boldsymbol{x}_0]\Delta\boldsymbol{x} \\ &\quad -\nabla\boldsymbol{F}[\boldsymbol{x}_0]^\top \boldsymbol{C}_{ss}^{-1}\left(\boldsymbol{s}-\boldsymbol{F}[\boldsymbol{x}_0]\right) \end{aligned} \tag{11}$$

Setting those derivatives equal to zero and solving for $\Delta\boldsymbol{x}$ yields the displacement

$$\Delta\boldsymbol{x} = \left(\nabla\boldsymbol{F}[\boldsymbol{x}_0]^\top \boldsymbol{C}_{ss}^{-1}\nabla\boldsymbol{F}[\boldsymbol{x}_0]\right)^{-1} \nabla\boldsymbol{F}[\boldsymbol{x}_0]^\top \boldsymbol{C}_{ss}^{-1}\left(\boldsymbol{s}-\boldsymbol{F}[\boldsymbol{x}_0]\right) \tag{12}$$

Although this can be seen as an iterative localisation algorithm in its own right we are more interested in the error propagation from the signal level $\boldsymbol{s}$ to the unknown location $\boldsymbol{x}$. From the linearity of the last equation follows that the expected covariance matrix of the resulting position estimate can be derived using linear error propagation as follows (cf. [21])

$$\begin{aligned} \boldsymbol{C}_{xx} &= \left(\nabla\boldsymbol{F}[\boldsymbol{x}_0]^\top \boldsymbol{C}_{ss}^{-1}\nabla\boldsymbol{F}[\boldsymbol{x}_0]\right)^{-1} \nabla\boldsymbol{F}[\boldsymbol{x}_0]^\top \boldsymbol{C}_{ss}^{-1} \\ &\quad \boldsymbol{C}_{ss}\boldsymbol{C}_{ss}^{-1}\nabla\boldsymbol{F}[\boldsymbol{x}_0]\left(\nabla\boldsymbol{F}[\boldsymbol{x}_0]^\top \boldsymbol{C}_{ss}^{-1}\nabla\boldsymbol{F}[\boldsymbol{x}_0]\right)^{-1} \end{aligned} \tag{13}$$

$$= \left(\nabla\boldsymbol{F}[\boldsymbol{x}_0]^\top \boldsymbol{C}_{ss}^{-1}\nabla\boldsymbol{F}[\boldsymbol{x}_0]\right)^{-1} \tag{14}$$

Note that the expected localisation accuracy does not depend on the measured signal strengths but solely depends on the fingerprint map and can therefore be derived in advance for every location $\boldsymbol{x}_0$ based on the fingerprints alone yielding a corresponding accuracy map that can be used to assess the expected localisation accuracy for each position.

In order to capture the resulting localisation accuracy in a single figure we will use the square-root of the largest eigenvalue of $\boldsymbol{C}_{xx}$, which is the expected localisation standard deviation in the direction of maximum uncertainty. In the following section we will give an example how the presented technique can be applied to assess the localisation accuracy of a given WiFi installation.

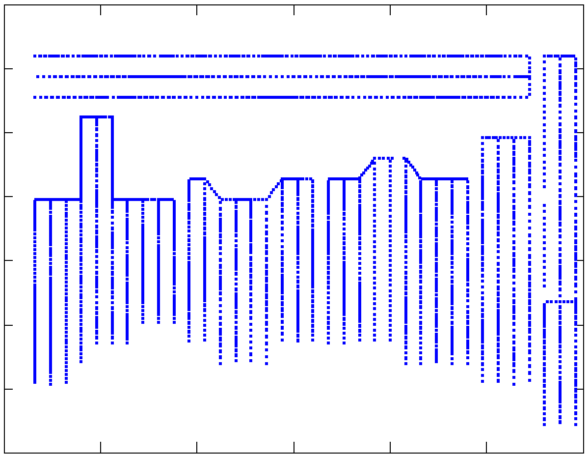

Fig. 1. Sampling grid used to measure the fingerprint map in a big hall 30×60 meters in size. To the top there is a balcony and to the left there is an escalator connecting the ground floor with the balcony.

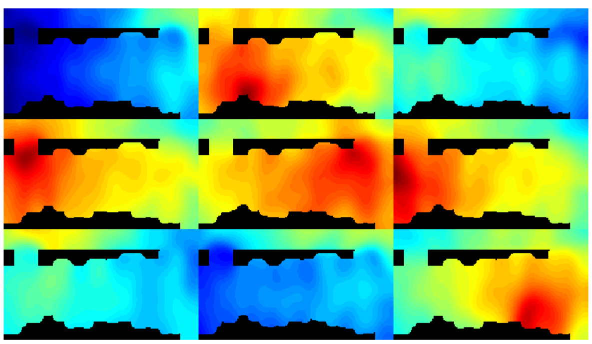

Fig. 2. Smoothed WiFi RSSI fingerprint map comprising of 9 access points in a big hall 30×60 meters in size. To the top there is a balcony and to the left there is an escalator connecting the ground floor with the balcony. The smoothing kernel size used was $\sigma = 2m$, the values range from $-80dBm$ (blue) to $-35dBm$ (red).

IV. Example

In order to illustrate the usefulness of the algorithm outlined in the previous section we will show its results for an existing WiFi installation covering a large hall of $30 \times 60m^2$ with 9 access points comprising of a ground level, an escalator to the left, and a balcony in the top part of the map.

Figure 1 shows the sampling points where the signal strengths were measured in order to derive a fingerprint map. The measurement was carried out by walking up and down along a pre-defined path and interpolating the positions in between while recording RSSI values for each of the access points received. Figure 2 shows the resulting smoothed fingerprint maps using a smoothing kernel size of $\sigma = 2m$. You can see that the access points are more or less evenly distributed across the environment.

Figures 3 and 4 show the gradient fingerprint map in x- and y-direction respectively. As the localisation accuracy basically depends on sums of the product of those two, accurate positioning is only possible where these gradients are both significantly different from zero for at least one access point.

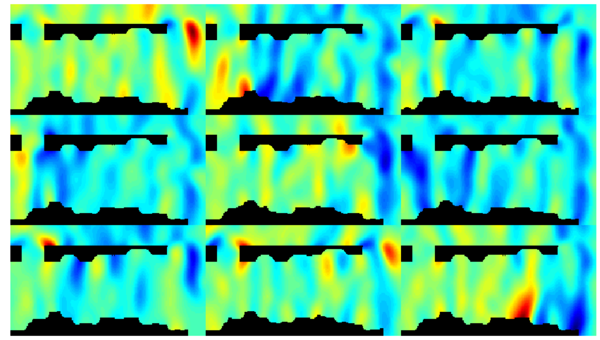

Fig. 3. Gradient fingerprint map in the direction of the x-axis (blue = negative, red = positive).

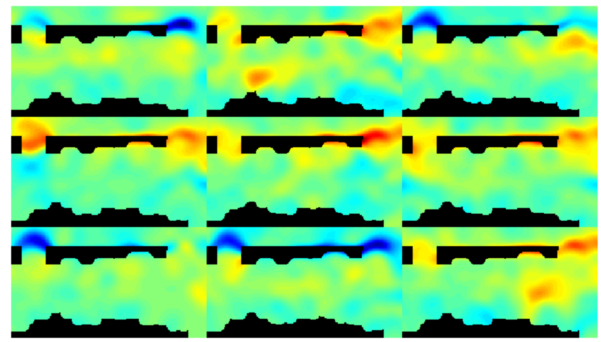

Fig. 4. Gradient fingerprint map in the direction of the y-axis (blue = negative, red = positive).

Figure 5 finally shows the resulting accuracy map. The red areas indicate where only poor localisation accuracy of about $15m$ per dBm RSSI measurement accuracy is achievable, while the blue areas allow for an expected localication accuracy in the range of $2m$ per dBm RSSI measurement accuracy.

Comparing this localisation accuracy map with the coverage map depicted in figure 6 or the signal-to-noise ratio map depicted in figure 7 you can see, that although in general one can say that localisation accuracy is worst in areas of low signal level and signal-to-noise ratio (where the fingerprint

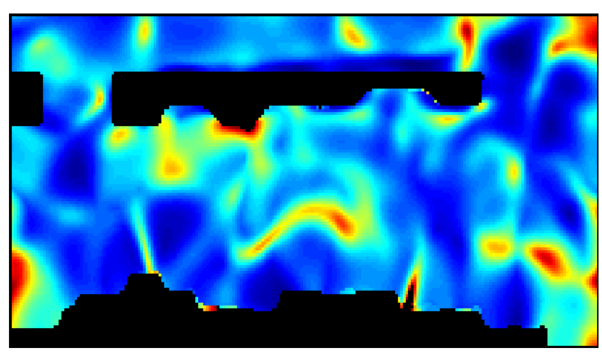

Fig. 5. Square-root of the largest eigenvalue of the expected localisation uncertainty in each point. The values range from $2m$ per dBm RSSI measurement accuracy (blue) up to $15m$ per dBm RSSI measurement accuracy (red).

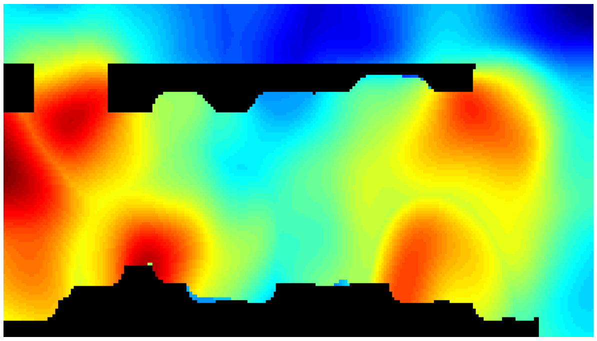

Fig. 6. WiFi coverage in terms of maximum signal level from every access point in each point.

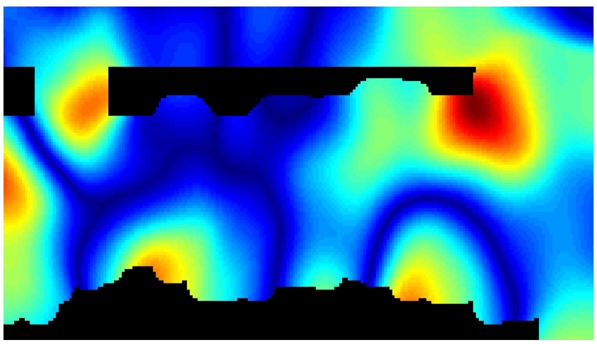

Fig. 7. Signal to noise ratio, i.e. the ratio between the maximum signal level and the sum of all the other signal levels, in each point.

map is in general quite flat and therefore has no significant gradient), the expected localisation accuracy behaves rather different to those other two measures used for optimising WiFi designs. This suggests that when designing access point layouts special care should be taken to maximise localisation accuracy as well if a WiFi based localisation system is to be installed.

V. Conclusion and Outlook

We have presented a method for the prediction of expected localisation accuracy based on the analysis of the maximum likelihood estimate of the position from a signal strength measurement. Assuming a Gaussian error model it has been shown how this accuracy solely depends on the gradient fingerprint map. We have also shown how this gradient fingerprint map is efficiently computable in advance from given fingerprint data using standard image processing techniques. The prediction of expected accuracy allows to see in advance how fingerprinting based localisation algorithms will behave in terms of achievable accuracy and therefore enables the design of WiFi access point layouts also based on localisation needs rather than solely based on coverage or signal-to-noise criteria.

The integration of localisation accuracy criteria into WiFi design tools promises to increase the overall accuracy of WiFi based localisation techniques from a practical point of view as much as improving the localisation algorithms themselves and will therefore be a promising direction for future research.

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