

# Three-Dimension Localization Method based on Time Difference Of Arrival for Ultra Wide Band Systems

Kobenan Ignace KOSSONOU\*, Yassin EL HILLALI\*, Michael BOCQUET\*, Jamal ASSAAD\*, Atika RIVENQ\*, Issa DOUMBIA\*\*

\* Institut d'Electronique, de Microélectronique et de Nanotechnologie - Département Opto-Acousto-Electronique (IEMN-DOAE), Université de Valenciennes et du Hainaut-Cambrésis (UVHC), Le Mont Houy, 59313 Valenciennes cedex 9 France. Email: [Kobenan.Kossonou@meletu.univ-valenciennes.fr](mailto:Kobenan.Kossonou@meletu.univ-valenciennes.fr)

\*\* Université de Cocody BP V34 Abidjan, Côte d'Ivoire.

**Abstract-** Several techniques have been developed for position location in a two dimension (2-D) system but few techniques have been done in a three dimension (3-D) context. Some of the techniques and algorithms proposed for 3-D are often subject to accuracy problems. We propose in this paper a novel technique of 3-D indoor position location. This method relies on techniques for ultra wide band (UWB) transmissions. The location algorithm is in terms of time difference of Arrival (TDOA). Numerical simulations show that the proposed approach improves precision of 3-D position location.

**Keywords—** TDOA; 3-D Position Location; UWB.

## I. INTRODUCTION

In general, finding with accuracy the position location of sensor in a 3-D system isn't easy due to the complexity of the resolution of the hyperbolic equations. Specifically, this accuracy is more difficult in indoor environment with the multiple Non Line Of Sight (NLOS). The feasibility studies of a 3-D position location scheme for example in indoor environments have been done in [1] and [2]. Some research works shown that there are errors in the results following to the third component (z). In Ref. [3], it has been suggested to spread the 3-D study in order to better characterize the impact of the component (z) on the accuracy of position location. Several techniques have been developed for positions locating in a 2-D but few of them have been done in a 3-D context. Furthermore, some works have shown that the iterative method gave better accuracy for 3-D position localization [2].

The purpose of this paper is to propose a method which gives better accuracy for a 3-D localization. Ref. [4] proposed this algorithm for a 2-D position location; we extend this algorithm firstly for a 3-D position location and secondly, we propose an approach that provides better estimate. To validate this method in a 3-D context, we consider simulated TDOA with errors and estimate the position of the sensor. Because UWB occupies large bandwidth and uses extra low power for transmission, we use UWB Impulse Radio with Time Hopping Pulse Position Modulation (IR-UWB TH-PPM) signal to estimate TDOAs. We added error ranging from -0.06 to 0.05 ns on the TDOAs. We considered A White Gaussian Noise (AWGN) channel.

The remainder of the paper is organized as follows. In section 2, we describe the TDOA position location technique. Section 3 presents mathematical model for hyperbolic TDOA equations. In section 4, localization algorithm used to estimate the sensor position is presented. Simulation results are presented and analyzed in section 5. Finally, we conclude the paper in section 6.

## II. TDOA TECHNIQUE

In most cases, TDOA techniques are based on estimating the difference in the arrival times of the signal from the source at multiple receivers. In our study, we have one sensor and five sources. The scenario that we consider is such that the sensor calculates its own position. So it receives signal from the transmitters and the first signal that it detects is the reference signal. The received signal at the sensor is the superposition of the emitted signals. The cross-correlation of the received signal with each transmitted signal is done and the peak of the cross-correlation output gives the time delay for this signal. The difference of the peaks of the cross-correlation between the received signal and the first signal arrived on one hand and the following signal on the other hand represents the TDOA between the first transmitter and the following transmitter signal.

In a 2-D space, all possible positions of the sensor for same TDOA on a pair of transmitter is a hyperbola. And in a 3-D context, this set of possible positions of the sensor is a hyperboloid. So, each estimated TDOA defines a hyperboloid between the two transmitters on which the sensor may exist. To avoid raise any ambiguity on the issue, we need at least four transmitters in order to finally find the position of the sensor in a 3-D environment. In this paper, we consider five transmitters to obtain four TDOAs.

## III. MATHEMATICAL MODEL OF HYPERBOLIC EQUATIONS

This section describes the general location 3-D method. In our system, we have considered a limited set of fixed nodes which represent transmitters. There are five of them and they are positioned at known coordinates  $(x_i; y_i; z_i)$ . One of them is the reference. It is selected according to the time delay its signal is detected by the sensor. The sensor moves and its coordinates  $(x; y; z)$  are unknown.

The squared range distance ( $d_i$ ) between the sensor and the  $i^{\text{th}}$  transmitter is given by the relationship (1):

$$(d_i)^2 = (x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2. \quad (1)$$

Let ‘r’ be the letter which indicates the number of the reference transmitter, ( $t_i$ ) the time at which the  $i^{\text{th}}$  transmitter’s signal is detected, ( $t_r$ ) this one (time) of the reference, ( $c$ ) the speed of light and ( $d_{i,r}$ ) the range distance between the sensor and the reference transmitter. The range difference between the reference and the others transmitters is given by (2):

$$\begin{aligned} d_{i,r} &= d_i - d_r = c(t_i - t_r) \\ &= \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} \\ &\quad - \sqrt{(x_r - x)^2 + (y_r - y)^2 + (z_r - z)^2}. \end{aligned} \quad (2)$$

where ( $t_i - t_r$ ) is the TDOA between the emitter (i) and emitter (r).

Equation (2) defines the set of nonlinear hyperbolic equations whose solution gives 3-D coordinates of the sensor. Solving them is difficult. Consequently, linearizing this set of equations is commonly performed. One way of linearizing these equations is done through the use of Taylor-series expansion and retaining the first two terms [5]. A commonly used alternative method to the Taylor-series expansion method is to first transform the set of nonlinear equations in (2) into another set of equations [6], [7].

Rearranging the form of (2) into

$$(d_i)^2 = (d_{i,r} + d_r)^2. \quad (3)$$

Equation (1) can now be written as:

$$\begin{aligned} (d_{i,r})^2 + 2d_{i,r}d_r + (d_r)^2 &= (x_i)^2 + (y_i)^2 + (z_i)^2 - 2x_i - 2y_iy \\ &\quad - 2z_i z + x^2 + y^2. \end{aligned} \quad (4)$$

Subtracting (1) at  $i = r$  from (4), results in

$$\begin{aligned} (d_{i,r})^2 + 2d_{i,r}d_r &= (x_i^2 + y_i^2 + z_i^2) - (x_r^2 + y_r^2 + z_r^2) \\ &\quad - 2(x_i - x_r)x - 2(y_i - y_r)y - 2(z_i - z_r)z. \end{aligned} \quad (5)$$

Let be  $K_i = x_i^2 + y_i^2 + z_i^2$ ,  $K_r = x_r^2 + y_r^2 + z_r^2$ ;

$$x_{i,r} = x_i - x_r; \quad y_{i,r} = y_i - y_r \quad \text{and} \quad z_{i,r} = z_i - z_r.$$

Then, equation (5) becomes

$$(d_{i,r})^2 + 2d_{i,r}d_r = K_i - K_r - 2x_{i,r}x - 2y_{i,r}y - 2z_{i,r}z. \quad (6)$$

The set of equations in (6) are now linear with the sensor coordinates ( $x$ ;  $y$ ;  $z$ ) and the range of the reference emitter to the sensor ( $d_r$ ) as the unknowns, and more easily handled.

In the literature, to solve this set of equations, two situations have to be considered: where the emitters are arranged linearly and where they are distributed arbitrarily.

When the emitters are arranged linearly, the estimation of the position location of the sensor is simplified. The methods which describe this situation for 2-D location position with an exact solution exist in [4].

The situation which is more complex is when the emitters are distributed arbitrarily. Some methods have been already proposed, but they are limited. In the following section, we describe the method developed in [4] and we add some hypothesis for our study.

#### IV. METHOD DESCRIPTION

We considered for our work that the emitter’s number is fixed to five and one sensor. Obviously, the method can be developed for higher number of transmitters.

The set of equations described in (6) are transformed as follows. On one side, we put the known parameters and on the other side the unknown such as:

$$\frac{1}{2}(d_{i,r}^2 - K_i + K_r) = -(x_{i,r}x + z_{i,r}y + z_{i,r}z + d_{i,r}d_r). \quad (7)$$

This equation is the  $i^{\text{th}}$  line of matrix with (M-1) rows where M indicates total number of transmitters. This line can be written as:

$$\frac{1}{2}(d_{i,r}^2 - K_i + K_r) = -[x_{i,r} \quad z_{i,r} \quad z_{i,r} \quad d_{i,r}] \begin{bmatrix} x \\ y \\ z \\ d_r \end{bmatrix} \quad (8)$$

where  $r \in \{1, 2, 3, \dots, M\}$  and  $i = (1, 2, 3, \dots, M) \setminus \{r\}$ . For example, if the number of emitters is five and the reference emitter is the number five, then  $r = 5$  and  $i = 1, 2, 3, 4$ .

When TDOA estimated values are exact, the above result of the left side members will be identical to those on the right side. Generally, these values are never exact. They are often estimated with some errors. We indicate those error vectors by ( $W$ ), the left side matrix by ( $h$ ); the right side matrix by ( $G_a$ ) and ( $Z_a$ ) respectively. So (8) becomes:  $h = G_a Z_a$ . Then we obtain (9):

$$W = h - G_a Z_a. \quad (9)$$

The elements of  $Z_a$  are related to (1), which means that (9) is still a set of nonlinear equations in three variables  $x$ ,  $y$ , and  $z$ . In [4], the approach used to solve the nonlinear problem is the two-stage maximum likelihood (ML) method. They assume firstly that there is no relationship among the coordinates of the sensor and the range distance between the sensor and the reference emitter. Weighted linear Least-Squares (LS) gives an initial solution. A second weighted LS, done by imposing the known relationship (1) to the initial solution gives an improved final solution of position estimate.

##### A. First Estimation

In this paper, we assume general case where emitters and sensor are far away from each other. Because of that, we use the equation (10) which contains the elements of the first estimation.  $Q$  is the covariance matrix of TDOAs vector. Its size is (M-1) x (M-1),  $a^2$  for diagonal elements and  $0.5a^2$  for all other elements, where  $a^2$  is the TDOA variance. The first estimation solutions are given in [4]:

$$Z_{a1} = ((G_a)^T Q^{-1} G_a)^{-1} (G_a)^T Q^{-1} h. \quad (10)$$

We assume that ( $Z_{a,x}$ ;  $Z_{a,y}$ ;  $Z_{a,z}$ ;  $Z_{a,d_r}$ ) are initial solutions given by (10).

##### B. Second Estimation

To determine the final solution, new matrix and vectors are defined. This relationship takes into account the fact sensor coordinates and the estimated range distance between the sensor and the reference. The errors vector over this can be rewritten as:

$$W_2 = h_2 - G_{a2} Z_{a2} \quad \text{With:}$$

$$h_2 = \begin{bmatrix} (Z_{a,x} - x_r)^2 \\ (Z_{a,y} - y_r)^2 \\ (Z_{a,z} - z_r)^2 \\ (Z_{a,dr})^2 \end{bmatrix}; G_f = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}; Z_{a2} = \begin{bmatrix} (x - x_r)^2 \\ (y - y_r)^2 \\ (z - z_r)^2 \end{bmatrix}. \quad (11)$$

Another matrix defined is the diagonal matrix which contains the estimated position with respect to the reference:

$$D = \text{diag}\{(Z_{a,x} - x_r), (Z_{a,y} - y_r), (Z_{a,z} - z_r), (Z_{a,dr})\}. \quad (12)$$

The final solution is derived from:

$$Z_f = (G_f^T D^{-1} Q^{-1} G_a D^{-1} G_a G_f)^{-1} (G_f^T D^{-1} G_a Q^{-1} G_a D^{-1}) h_2. \quad (13)$$

The final position estimate is then obtained from  $(Z_f)$  as

$$Z_p = \sqrt{Z_f} + \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} \text{ or } Z_p = -\sqrt{Z_f} + \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix}. \quad (14)$$

Solutions described by (14) are 3-D solutions like those for 2-D in [4]. When applying the method as described in [4] for 3D position location, (14) doesn't always give the best estimate; we get sometimes different results in two cases. The first case is when the sensor position is such that all its coordinates are greater than those of the reference transmitter. In this case, the estimates are given by one of the solutions given by (14).

The other case occurs when the sensor is posted or located such that one or two of its coordinates are smaller than those of the reference transmitter. In this case, none of the solutions proposed in (14) gives a good estimate. However, one of these solutions gives two coordinates which are exact.

To overcome this problem, we propose a new approach that corrects these errors.

### C. Performed Method For 3-D Position Location

Let  $(Z_{f,x}, Z_{f,y}, Z_{f,z})$  be the obtained solutions from (13). We assume that each coordinate can be positive or negative which results in two states. Given that, we defined  $2^3 = 8$  possible positions described as follows:

$$\begin{aligned} S_1 &= (\sqrt{Z_{f,x}} + x_r; \sqrt{Z_{f,y}} + y_r; \sqrt{Z_{f,z}} + z_r) \\ S_2 &= (-\sqrt{Z_{f,x}} + x_r; \sqrt{Z_{f,y}} + y_r; \sqrt{Z_{f,z}} + z_r) \\ S_3 &= (\sqrt{Z_{f,x}} + x_r; -\sqrt{Z_{f,y}} + y_r; \sqrt{Z_{f,z}} + z_r) \\ S_4 &= (\sqrt{Z_{f,x}} + x_r; \sqrt{Z_{f,y}} + y_r; -\sqrt{Z_{f,z}} + z_r) \\ S_5 &= (-\sqrt{Z_{f,x}} + x_r; -\sqrt{Z_{f,y}} + y_r; \sqrt{Z_{f,z}} + z_r) \\ S_6 &= (-\sqrt{Z_{f,x}} + x_r; \sqrt{Z_{f,y}} + y_r; -\sqrt{Z_{f,z}} + z_r) \\ S_7 &= (\sqrt{Z_{f,x}} + x_r; -\sqrt{Z_{f,y}} + y_r; -\sqrt{Z_{f,z}} + z_r) \\ S_8 &= (-\sqrt{Z_{f,x}} + x_r; -\sqrt{Z_{f,y}} + y_r; -\sqrt{Z_{f,z}} + z_r). \end{aligned} \quad (15)$$

These solutions include two solutions ( $S_1$  and  $S_8$ ) derived from equation (14). For each position ( $S_j$ ), we estimate another  $(d_{j,r})$ , with  $j=1, 2, \dots, 8$  and for each position, the  $(d_{j,r})$  is compared to  $(d_{i,r})$ . The position for which the  $(d_{j,r})$  are the closest to the  $(d_{i,r})$  will be the best estimate. In other words, we compare TDOAs of new estimated positions with actual TDOA. Finally, we select the one for which the TDOAs are the closest to the first

estimated TDOAs ( $t_i - t_r$ ). This technique automatically eliminates solutions which are not in the region of interest.

## V. SIMULATION RESULTS

The performance of this method is investigated through the use of computer simulations. Five emitters are used in the location system. Their coordinates are:

$E_1(5; 10; 7); E_2(-17; 8; 9); E_3(11; 14; -5); E_4(21; 11; 13); E_5(7; 2; 10)$ . The measurement unit is in meters (m).

Signals used for simulations are derived from the first five Gaussian pulses which were modulated in PPM with multiple access codes TH. Each transmitter uses pulse which corresponds to its number. We analyzed two cases; first case, where we add the same errors to the simulated TDOA errors ranging from 0.01 to 0.05 (ns) with a 0.001 step. In either case, the TDOA added noise differs from one transmitter to the other. These errors are distributed according to a pseudo-random normal distribution.

We suppose the sensor moves and we estimate its position in different positions. We present in this paper two positions:  $P_1(12; 7; 15)$  and  $P_2(8; 22; 2)$ . For each position, we realized 1000 runs and determinate the number of reference transmitters and the number of solutions for which the estimate is the best. Finally, we plot the Root Mean Squared Error (RMSE) for different positions as a function of Signal to Noise Ratio (SNR) ranging from -10 dB to 10 dB.

### A. Addition Of Same Errors On TDOA Estimate

We can see in Fig. 1 of the zoomed positions coordinates between the actual and estimated sensor at  $P_1(12; 7; 15)$  and  $P_2(8; 22; 2)$ . Their RMSE plots (RMSE- $P_1$ , RMSE- $P_2$ ) are given in Fig. 2. All estimated positions that we proposed are also presented in Table I.

For the first position, simulations show that the reference emitter is the fifth ( $E_5$ ) and  $S_1$  is the best estimation among the eight solutions we propose. Sensor position is estimated at (12.004; 6.979; 14.996). TDOA RMSE (RMSE- $P_1$ ) is constant at 0.0137m. For second case we analyze in our works,  $S_2$  gives the best estimate and the reference emitter is the third ( $E_3$ ). Sensor position is then located at (8.003; 22.002; 1.985). We can remark also that,  $S_1$  one of two solutions in (14), gives two exact coordinates (y and z) and the solution  $S_8$  gives wrong estimate. The RMSE (RMSE- $P_2$ ) is 0.0100m.

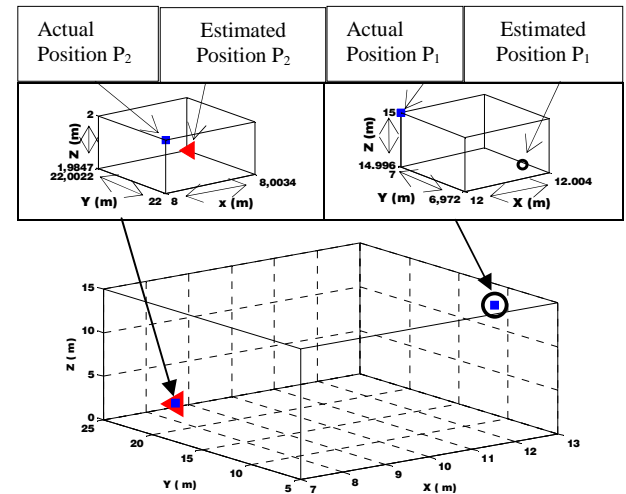


Figure 1. Actual and Estimated Positions  $P_1$  and  $P_2$

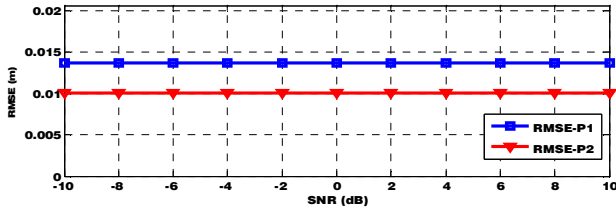


Figure. 2 RMSE in AWGN channel with same TDOA errors

TABLE I. EIGHT PROBABLE POSITIONS FOR SENSOR LOCATED AT  $P_1(12; 7; 15)$  AND  $P_2(8; 22; 2)$

	$P_1(12; 7; 15)$			$P_2(8; 22; 2)$		
	X(m)	Y(m)	Z(m)	X(m)	Y(m)	Z(m)
S1	12.004	6.979	14.996	13.997	22.002	1.985
S2	01.997	6.979	14.996	8.003	22.002	1.985
S3	12.004	-2.979	14.996	13.997	5.998	1.985
S4	12.004	6.979	5.004	13.997	22.002	-11.985
S5	01.997	-2.979	14.996	8.003	5.998	1.985
S6	01.997	6.979	5.004	8.003	22.002	-11.985
S7	12.004	-2.979	5.004	13.997	5.998	-11.985
S8	01.997	-2.979	5.004	8.003	5.998	-11.985

B. Addition Of Different Errors On TDOA Estimate

As shown in the first case, illustrations of the second case are shown in Fig. 3 and Fig. 4. Estimated positions are given in Table II. As previously indicated, same results were obtained for each position. However, RMSE increased with the added error. In this case, the added error ranges from -8.857 ns to +5.926 ns. TDOA RMSE of position  $P_1$  (RMSE- $P_1$ ) ranged from 0.02m to 0.03m, and this of position  $P_2$  (RMSE- $P_2$ ) ranged from 0.06m to 0.08m.

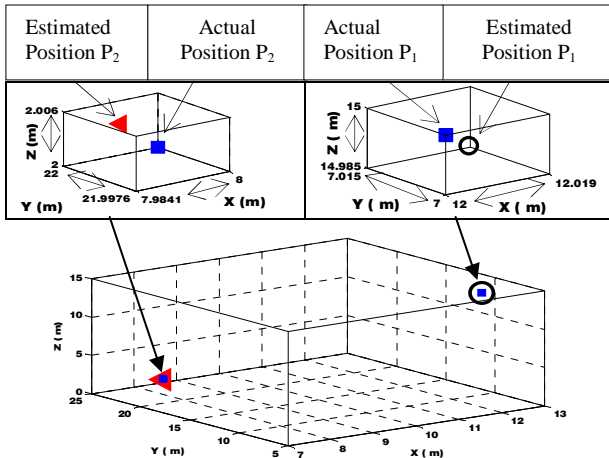


Figure. 3 Actual and Estimated Positions  $P_1$  and  $P_2$

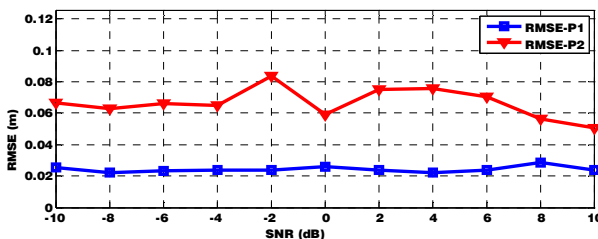


Figure. 4 RMSE in AWGN channel with different TDOA errors

TABLE II. EIGHT PROBABLE POSITIONS FOR SENSOR LOCATED AT  $P_1(12; 7; 15)$  AND  $P_2(8; 22; 2)$

	$P_1(12; 7; 15)$			$P_2(8; 22; 2)$		
	X(m)	Y(m)	Z(m)	X(m)	Y(m)	Z(m)
S1	12.019	7.015	14.985	14.026	21.998	2.006
S2	1.981	7.015	14.985	7.984	21.998	2.006
S3	12.019	-3.037	14.985	14.026	6.053	2.006
S4	12.019	7.015	5.039	14.026	21.998	-12.063
S5	1.981	-3.037	14.985	7.984	6.053	2.006
S6	1.981	7.015	5.039	7.984	21.998	-12.063
S7	12.019	-3.037	5.039	14.026	6.053	-12.063
S8	1.981	-3.037	5.039	7.984	6.053	-12.063

Results confirm our hypothesis: indeed in the first position considered, all the coordinates of the reference transmitter are much higher than those of the sensor. Therefore, the first solution gives the best estimate. In other case, when one component of the sensor is lower than the reference, the first or eighth solution could not provide good estimates. With this approach, results are the same when transmitters are near or far away from the sensor and RMSE is better than [4].

VI. CONCLUSION

In this paper, localization method for ultra-wideband (UWB) system is studied. The method used in the localization algorithm is in TDOA terms. Simulations results achieved allowed us to validate our hypothesis. This approach improves the method described in [4] and gives another method for 3-D localization with more accuracy. However, several research challenges remain to be investigated or addressed. Experimentation in realistic settings (in the presence of physical obstacles such as NLOS) is essential in evaluating the performance of the localization algorithms under consideration.

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