Pulsed Pseudolite Signal Effects on Non-Participating GNSS Receivers

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***Abstract*—Pseudolites are a technology with the potential of bridging the gap between outdoor and indoor navigation. Despite their potential, pseudolites can cause severe interference problems with non-participating receivers, i.e., devices not designed to exploit this technology.**
In this paper, the loss caused by pulsed pseudolite signals is determined as a function of the pulse duty cycle and the number of bits employed for signal quantization. Quantization, blanking and noise increase are identified as the main sources of signal degradation. Theoretical results are validated by simulations and experiments performed using commercial GPS receivers. The good agreement between theoretical and experimental results supports the validity of the proposed approach.

***Index Terms*—Global Navigation Satellite Systems, GNSS, non-participating receivers, pseudolite, quantization**

I. Introduction

Pseudolites or pseudo-satellites are an emerging technology that has the potential to extend the capability of Global Navigation Satellite Systems (GNSS) indoors and in harsh environments where GNSS services are denied. Despite their potential, pseudolites could cause severe interference problems to non-participating receivers, i.e., GNSS receivers unable or not designed to use pseudolite signals.

In this paper, a theoretical framework able to quantify the impact of pulsed pseudolite signals on non-participating receivers is developed. The model allows one to quantify the post-correlation loss as a function of the pulse duty cycle, the Automatic Gain Control (AGC) gain and the number of bits used for signal quantization. Quantization, blanking and noise increase are identified as the main sources of signal degradation. The proposed model extends previous results [1] that neglected the impact of the receiver front-end.

Monte Carlo simulations and experiments involving commercial GPS receivers are used to corroborate the validity of the developed model. The good agreement between theoretical, simulation and experimental results supports the effectiveness of the developed analytical framework.

II. Signal and System Model

The signal at the input of a GNSS receiver in a one-path additive Gaussian channel and in the presence of pseudolite signals can be modeled as

$$r(t) = \sum_{l=0}^{L-1} y_l(t) + \sum_{j=0}^{J-1} s_{p,j}(t) p_j(t) + \eta(t) \tag{1}$$

which is the sum of L useful signals transmitted by L different satellites, J pseudolite signals, and a noise term, $\eta(t)$. Each useful signal, $y_l(t)$ can be expressed as

$$y_l(t) = \sqrt{2C_l} d_l \left(t - \tau_{0,l}\right) c_l \left(t - \tau_{0,l}\right) \cos\left(2\pi(f_{RF} + f_{d,l})t + \varphi_{0,l}\right) \tag{2}$$

where

- C_l is the power of the lth useful signal;
- $d_l(\cdot)$ is the navigation message;
- $c_l(\cdot)$ is the lth pseudo-random sequence extracted from a family of quasi-orthogonal codes and used for spreading the signal spectrum
- $\tau_{0,l}$, $f_{d,l}$ and $\varphi_{0,l}$ are the delay, Doppler frequency and phase introduced by the communication channel
- f_{RF} is the centre frequency of the GNSS signal.

In (1), $s_{p,j}(t)p_j(t)$ denotes the jth pseudolite signal that has been modeled as the product of two terms: $s_{p,j}(t)$ is a continuous signal that assumes the same form as (2) and $p_j(t)$ is a binary sequence assuming values in $\{0, 1\}$ and defining a pulsing sequence. $p_j(t)$ defines the time instants when the pseudolite signal is effectively transmitted. The choice of this pseudolite signal format is common in the literature [1], [2] and justified by fact that it implies minimal firmware changes for enabling pseudolite processing in current GNSS receivers. The pulsing scheme, $p_j(t)$, has been introduced as an effective technique for reducing interference problems [1] with non-participating receivers.

Due to the quasi-orthogonality of the spreading codes, a GNSS receiver is able to process the L useful signals independently. In addition to this, the case of a single pseudolite signal is considered and (1) becomes

$$r(t) = y(t) + s_p(t)p(t) + \eta(t) \tag{3}$$

where the indexes, l and j have been dropped for ease of notation.

Signal (3) is filtered and down-converted by the receiver front-end before being digitized. Digitization implies two different operations: sampling and amplitude quantization. In the following, sampling and quantization are considered separately. In addition, it is assumed that the signal is sampled without introducing significant distortions. More specifically, after down-conversion and sampling (3) becomes:

$$\begin{aligned} r_{IF}[n] &= y_{IF}(nT_s) + s_{p,IF}(nT_s)p(nT_s) + \eta_{IF}(nT_s) \\ &= y_{IF}[n] + s_{p,IF}[n]p[n] + \eta_{IF}[n] \end{aligned} \tag{4}$$

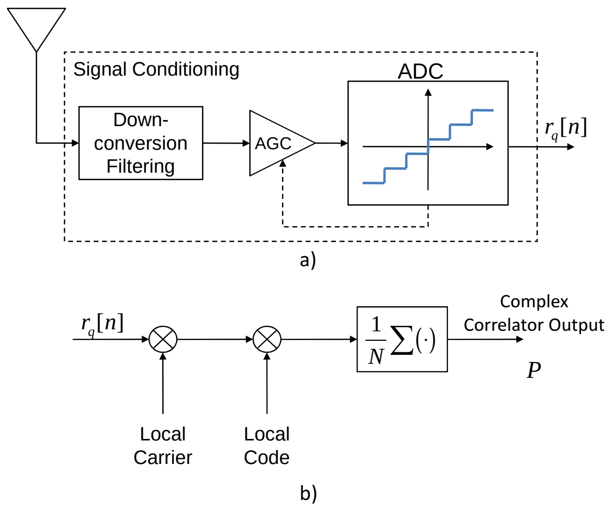

Fig. 1. Basic operations performed by a GNSS receiver. a) Signal conditioning: the RF analog signal is converted to an IF digital sequence. b) The correlation process.

where the notation $x[n]$ is used to denote a discrete time sequence sampled at the frequency $f_s = \frac{1}{T_s}$. The index "IF" is used to denote a signal down-converted to an intermediate frequency, f_{IF}. In (4),

$$\begin{aligned} y_{IF}[n] =& \sqrt{2C} d\left(nT_s - \tau_0\right) c\left(nT_s - \tau_0\right) \\ &\cdot \cos\left(2\pi\left(f_{IF} + f_0\right) nT_s + \varphi_0\right) \end{aligned} \tag{5}$$

and

$$\begin{aligned} s_{p,IF}[n] =& \sqrt{2C_p} d_p\left(nT_s - \tau_p\right) c_p\left(nT_s - \tau_p\right) \\ &\cdot \cos\left(2\pi\left(f_{IF} + f_p\right) nT_s + \varphi_p\right) \end{aligned} \tag{6}$$

where the subscript "p" is used to denote quantities relative to the pseudolite signal. The noise term, $\eta_{IF}[n]$, is assumed to be a white additive Gaussian noise with variance σ_{IF}^2. σ_{IF}^2 depends on the filtering, down-conversion and sampling strategy applied by the receiver front-end. A convenient choice is to assume $\sigma_{IF}^2 = N_0 B_{IF}/2$ where B_{IF} is the front-end bandwidth and N_0 is the power spectral density of the input noise $\eta(t)$. The ratio between the carrier power, C, and the noise power spectral density, N_0, defines the Carrier-to-Noise density power ratio, C/N_0, one of the main signal quality indicators used in GNSS.

The amplitude quantization process is shown in the upper part of Fig. 1 and consists of a scaling, introduced by the AGC, and a mapping of the continuous values of $y_{IF}[n]$ into a finite discrete alphabet. This second operation is performed by the Analog-to-Digital Converter (ADC). In this paper, ideal and very slow AGCs are considered. Definitions of different AGC types are provided by [1] and the considered cases are characterized by constant AGC gains. An ideal AGC ignores the pseudolite pulse and provides a gain independent from the pseudolite power. In the very slow case, the pseudolite power impacts the selection of the gain that is fixed according to the total power measured by the AGC. The case of a simple uniform quantizer is considered and it is assumed that the pseudolite power is sufficiently high to saturate the ADC. Under these conditions, the signal at the output of the ADC can be modeled as:

$$\begin{aligned} r_q[n] =& Q_B(A_g(y_{IF}[n] + \eta_{IF}[n]))(1 - p[n]) \\ &+ A_{max} \text{sign}\left(s_{p,IF}[n]\right) p[n] \end{aligned} \tag{7}$$

where

- $Q_B(\cdot)$ is the quantization function adopted by an ADC employing B bits for the signal representation. In a uniform quantizer, $Q_B(\cdot)$ is a symmetric stair-case function producing output values in the set $\left\{-2^B+1, -2^B+3, ..., -1, 1, ..., 2^B-3, 2^B-1\right\}$;
- A_g is the AGC gain;
- A_{max} is the maximum value representable by the ADC. In the uniform quantizer considered in this paper, $A_{max} = 2^B - 1$.

In (7), the pseudolite signal, $s_{p,IF}[n]$, is always saturating the ADC and thus it assumes values equal to $\pm A_{max}$. In addition to this, useful signal and noise components are effectively blanked by the pseudolite pulse justifying the presence of the $(1 - p[n])$ term in (7). Eq. (7) is the basic equation that will be used in the following to derive the expression of the loss caused by pulsed pseudolite signals.

A. The Correlation Process

After signal conditioning, the sequence $r_q[n]$ is correlated with local replicas of the signal code and carrier. This process is shown in the lower part of Fig. 1 and a complex correlator output is computed as

$$P = \frac{1}{N}\sum_{n=0}^{N-1} r_q[n] c\left(nT_s - \tau\right) \exp\left\{-j2\pi(f_{IF} + f_d) nT_s - j\varphi\right\} \tag{8}$$

where τ, f_d and φ are the code delay, the Doppler frequency and the phase tested by the receiver. N is the number of samples used for computing a correlator output and $T_c = NT_s$ is the coherent integraton time. It is noted that the computation of correlator outputs is essential for the proper functioning of a GNSS receiver and they are used both in acquisition and tracking [3], the main receiver operating modes. Thus, the quality of a GNSS signal can be defined after correlation as [4], [5]:

$$SNR_{out} = \max_{\tau, f_d, \varphi} \frac{\left|\text{E}\left\{P\right\}\right|^2}{\frac{1}{2}\text{Var}\left\{P\right\}} \tag{9}$$

where SNR_{out} is the coherent output Signal-to-Noise Ratio (SNR). The factor $1/2$ in (9) accounts for the fact that P is a complex quantity and only the variance of its real part is considered. The loss experienced at the correlator output due to the presence of a strong pulsed pseudolite signal is determined as the ratio between the measured SNR and the ideal coherent output SNR determined in the absence of quantization and interference.

III. Post-Correlation Loss

In order to compute the coherent output SNR in the presence of pseudolite pulses, it is necessary to characterize the different terms present in P. More specifically, the operations performed in (8) are linear and

$$P = P_q + P_s \tag{10}$$

where P_q is obtained by correlating the signal and noise term, $Q_B\left(A_g\left(y_{IF}[n] + \eta_{IF}[n]\right)\right)$, and P_s is determined by processing the pseudolite component. Using an approach similar to the one adopted by [6] for determining the quantization loss in the absence of pseudolite signal, it is possible to show that under perfect frequency and delay alignment

$$E\left[P_q\right] = \sqrt{\frac{C}{2\sigma_{IF}^2}} a(1-d)\exp\left\{j\Delta\varphi\right\} \tag{11}$$

where $\Delta\varphi$ is the residual phase difference between local and incoming signals,

$$a = \sqrt{\frac{2}{\pi}}\left[1 + 2\sum_{i=1}^{2^{B-1}-1}\exp\left\{-\frac{i^2}{2A_g^2\sigma_{IF}^2}\right\}\right] \tag{12}$$

and

$$d = E\left[\frac{1}{N}\sum_{i=0}^{N-1} p[n]\right] \tag{13}$$

is the duty cycle of the pseudolite pulsing scheme. It is noted that the blanking performed by the pseudolite pulse reduces the signal amplitude by a factor equal to $(1-d)$ whereas the factor a models the effect of AGC/ADC [6]. Perfect frequency and delay alignment are the results of the maximization process performed in (9). In a similar way,

$$\text{Var}\left\{P_q\right\} = \frac{b}{N}(1-d) \tag{14}$$

where

$$b = 1 + 8\sum_{i=0}^{2^{B-1}-1} i\text{erfc}\left(\frac{i}{\sqrt{2}A_g\sigma_{IF}}\right) \tag{15}$$

where $\text{erfc}(\cdot)$ denotes the complementary error function. It is noted that the quantity $L_q = \frac{a^2}{b}$ defines the quantization loss obtained in the absence of pseudolite signal.

P_s is obtained by correlating the term $A_{max}\text{sign}\left(s_{p,IF}[n]\right)p[n]$ with the locally generated code and carrier. Under the assumption that $\text{sign}\left(s_{p,IF}[n]\right)$ is uncorrelated with the local code, it is possible to show that P_s is zero mean and with variance

$$\text{Var}\left\{P_s\right\} = \frac{A_{max}^2}{N}d = \frac{\left(2^B-1\right)^2}{N}d. \tag{16}$$

Thus, assuming independence between noise and pseudolite components, the coherent output SNR in the presence of pseudolite signal becomes

$$\begin{aligned} SNR_{out} &= \frac{NC}{\sigma_{IF}^2}\frac{a^2(1-d)^2}{b(1-d) + \left(2^B-1\right)^2 d} \\ &= \frac{NC}{\sigma_{IF}^2}\frac{L_q\cdot(1-d)}{1 + g(B,A_g)\frac{d}{1-d}} \end{aligned} \tag{17}$$

where

$$g(B,A_g) = \frac{\left(2^B-1\right)^2}{1 + 8\sum_{i=0}^{2^{B-1}-1} i\text{erfc}\left(\frac{i}{\sqrt{2}A_g\sigma_{IF}}\right)} \tag{18}$$

defines the post-quantization Pseudolite-to-Noise Ratio (PNR). The ideal coherent output SNR is given by $SNR_{id} = \frac{NC}{\sigma_{IF}^2}$ and the loss due to quantization and pulsed interference becomes

$$\frac{L_q\cdot(1-d)}{1 + g(B,A_g)\frac{d}{1-d}}. \tag{19}$$

From (19), it is possible to note that the SNR is degraded by three factors:

- **quantization**: a quantization loss, L_q, is introduced independently from the presence of pulsed interference
- **blanking**: the pseudolite pulse effectively blanks the useful signal and noise components leading to an SNR reduction equal to $(1-d)$
- **noise increase**: at the correlator output, the pseudolite pulse is perceived as an additional noise term that is proportional to the post-quantization PNR and the relative duration of the pulse $d/(1-d)$.

It is noted that (19) significantly differ from previous attempts to quantify the degradation introduced by pulsed pseudolite signals [1]. Previous results where often provided without proof or neglecting the impact of the AGC/ADC.

Eq. (19) can be easily generalized to the case where several pulses arriving from different pseudolites reach the non-participating receiver antenna. In this case, d is replaced by the percentage of time the receiver is blinded by the pseudolite pulses.

IV. Simulation Results

In order to support the theoretical results provided in Section III, Monte Carlo simulations have been performed where noisy GPS L1 C/A signals have been jammed by a pulsed pseudolite signal. The simulation results are shown in Fig. 2 where the loss caused by a pulsed pseudolite signal is provided as a function of the number of bits and the pulse duty cycle. Small circles represent simulation results wheres lines have been obtained using (19). A good agreement between theoretical and simulation results is observed supporting the validity of the proposed analytical model. In Fig. 2, the case of an ideal AGC has been considered and its gain has been fixed according to the formula

$$A_g = \frac{1}{\sigma_{IF}}\sqrt{3^{(B-2)}} \tag{20}$$

that effectively interpolates the optimal AGC gain determined in [6]. When the pulse duty cycle tends to zero, the total loss tends to the quantization loss alone justifying the different intercepts with the y-axis. It is also noted that the curves in Fig. 2 have different slopes. This is due to the fact $g(B,A_g)$ increases with the number of bits when the (20) is valid. This is in agreement with the fact that multi-bit receivers are more sensitive to pulsed interference [1].

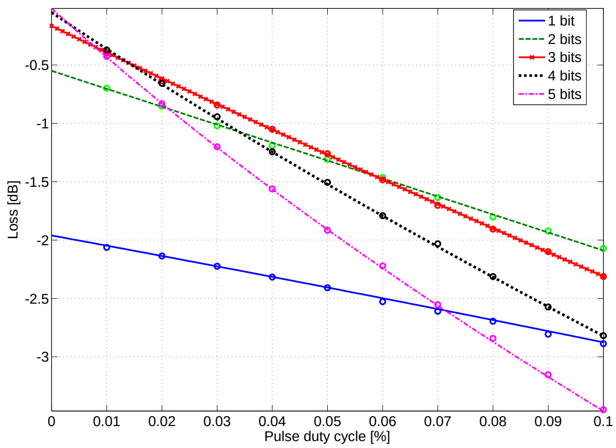

Fig. 2. Loss caused by a pulsed pseudolite signal as a function of the number of bits and the pulse duty cycle. Comparison between theoretical and simulation results. Small circles represent simulation results.

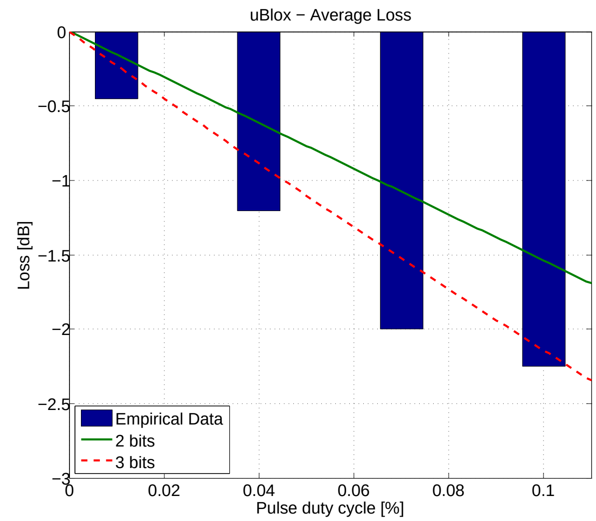

Fig. 3. Average C/N_0 loss measured for a uBlox 5 receiver in the presence of pulsed pseudolite signals. The quantization loss has been removed from the theoretical curves (2 and 3 bits).

V. Pseudolite Impact on Commercial Receivers

In order to further support the validity of the developed theory, a modified record and playback methodology has been developed for testing the impact of pulsed pseudolite signals on commercial GPS receivers. More specifically, live GPS data have been collected using a National Instruments (NI) PXIe-5663 vector signal analyzer. The NI PXIe-5663 allows one to produce a base-band digital representation of GPS signals that are stored in a binary file. A synthetic peudolite signal has been added to the base-band digital samples that have then been played back to RF using a NI PXIe-5450 signal generator. Experiments were performed in conducted mode using different commercial receivers. The loss induced by the psudolite signal has been evaluated as the difference between C/N_0 values estimated in the presence and absence of pseudolite signals. This approach allows one to measure only the loss due to blanking and noise increase. The quantization term in (19) is not observable since the signal at the input of the receiver is quantized independently from the pseudolite component.

Sample results obtained for a uBlox 5 receiver are shown in Fig. 3 where the C/N_0 loss is provided as a function of the duty cycle d. The measured loss is compared with the theoretical curves for the 2 and 3 bit cases. The quantization loss has been removed from the theoretical curves. The measured loss is close to the theoretical curves and a difference of about 0.5 dB is observed with respect to the 3 bit case. It is noted that when testing a commercial receivers, several parameters are not under the control of the experimenter and a perfect match between empirical and theoretical results cannot be expected. In this case, the number of bits used by the receiver and the algorithm employed to set the AGC threshold are not known. Thus, the theoretical curves used for the comparison have been computed assuming an ideal AGC. Despite these assumptions, the agreement between theoretical and empirical results shown in Fig. 3 support the validity of the developed theory and its ability to predict the performance of a commercial receiver in the presence of pulsed pseudolite signals.

VI. Conclusions

In this paper, a theoretical framework allowing the determination of the loss caused by pulsed pseudolite signals on non-participating GNSS receivers is provided. The developed theory significantly extends previous findings accounting for the presence of AGC and ADC and justifying, from an analytical point of view, several empirical results such as the higher vulnerability to pulsed interference of multi-bit GNSS receivers. Simulations and experiments performed using commercial GPS receivers supports the validity of the developed theory showing its capability of predicting the degradations caused by pulsed pseudolite signals.

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