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Accuracy enhancement in high sensitivity GNSS with Particle Filter approach

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***Abstract*— We propose a novel approach to estimate the position in weak signals conditions, based on Particle Filter algorithm. In urban canyoning and light indoor environment huge errors affect pseudorange measurements mainly due to multipath and thermal noise. In details in this work, we have empirically evaluated the statistical distribution of pseudo-ranges measurement error, taking into account heterogeneous environments. The error behavior may be described by Gaussian distribution, however long tails (outliers) are not taken into account by the Guassian approximation. The outliers may be isolated by integrity approach if redundancy is available. In high sensitivity environment the number of outliers and the system redundancy may allow to detect the presence of faulty measurements but not to isolate them. We propose under above conditions to estimate the position with particle filter approach, based on the empirical error distribution, achieving remarkable improvement in position accuracy.**

Keywords-component; Assisted-GNSS;High Sensitivity GNSS; Particle Filter;

I. INTRODUCTION

Assisted GNSS is deeply exploited as enabling technology for location based services, increasing receiver performance as time to first fix and sensitivity. In weak signals conditions, huge errors affect pseudorange measurements mainly due to multipath and thermal noise in the tracking loop. The received signal is distorted by many reflected components. Direct signals may be weaker than reflected/diffracted ones or may not be usable at all. A numerical error distribution of the measured pseudo-ranges may be estimated as a function of signal to noise ratio, the model is specific for each receiver. From the numerical data, we estimate the normal approximation of the error distribution [1]. We obtain similar results in error modelling for different propagation environments, in particular indoor and urban canyon scenarios, therefore a generic error model may be estimated merging data acquired in different propagation conditions. High sensitivity autonomous integrity monitoring algorithm [2] tests if the measurements errors belong to assumed Gaussian statistics. There are often highly perturbed measurements that do not belong to hypothesized normal statistics, if these outliers are not isolated the accuracy of WLS position estimation is poor. The outliers may be excluded only if the number of measurements is redundant with respect to the number of unknown variables. The paper proposes to estimate the receiver position implementing Particle Filter PF algorithm in cases when the fault measurements are detected but can not be isolated. We show that the novel algorithm provides for better accuracy than traditional WLS approach. The work has been developed using a Sirf StarIII high sensitivity receiver.

A brief description of the contents of this article is provided:

- First of all, we generate the numerical distribution function of error affecting the pseudorange measurements, taking into account different propagation scenarios. The Gaussian approximation is then estimated fitting the numerical distribution functions.
- Techniques for processing pseudorange measurements and estimate Position and Time (PT) parameters are presented, focusing on WLS approach and autonomous integrity testing technique.
- PF technique for PT estimation is described, based on the pseudorange error numerical distribution function
- A criterion based on integrity test is finally proposed for applying PF algorithm only in selected fixes. The benefit in terms of accuracy of the proposed approach is showed

II. PSEUDORANGE ERROR MODEL IN HIGH SENSITIVITY CONDITIONS

The acquisition of observation data for long time intervals at various sites permits to define an numerical model of the error affecting the pseudorange measurements, for the specific receiver [1]. Different propagation conditions have been taken into account in our work, merging data acquired in light indoor and urban canyoning sites. An empirical distribution function of the error is estimated, for each value of signal to noise ratio, as described in Fig 1.

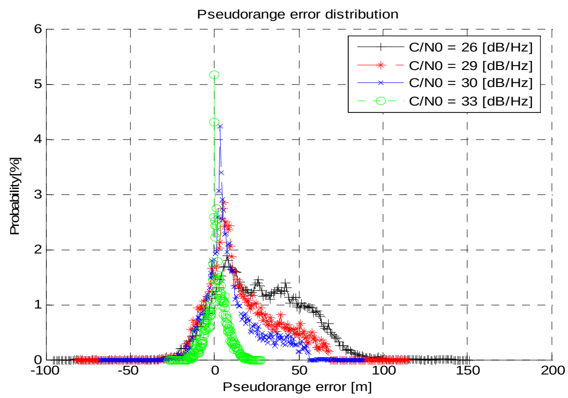

Figure 1. Generic pseudorange error distribution function.

This numerical error model can be considered as a generic one, applicable in any high sensitivity environments.

This numerical function can be fitted by a normal distribution: for each value of signal to noise ratio mean value and variance of Gaussian function are estimated. The error modelling methodology and results are described in [1]. Fig. 1 shows the observed Gaussian model (mean and variance).

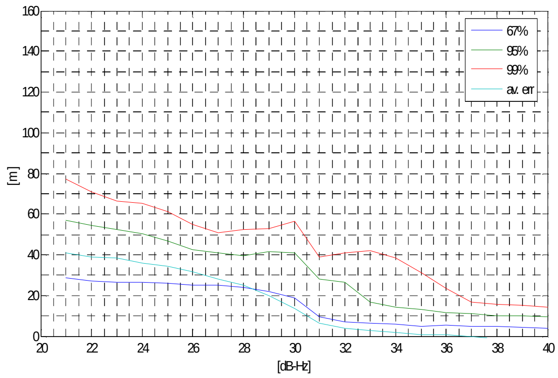

Figure 2. Indoor acquisition. Variance of pseudorange error [m] versus the signal to noise ratio [dB-Hz]. Also the average error versus the C/N0 is reported (light blue). The error percentiles are reported: 67% (blue), 95% (green) and 99% (red). Sirf starIII commercial receiver has been considered.

As depicted in Fig 1, the tails of the error distribution of pseudorange measurements are not Gaussians, due to a large amount of outliers in high sensitivity environments. In this work, we have consider an hybrid approach in error modeling, the distribution function is given by the combination of Gaussian function, for low confidence values (lower than 2σ), and for higher error values the empirical distribution, taking into account distribution long tails (Fig. 3).

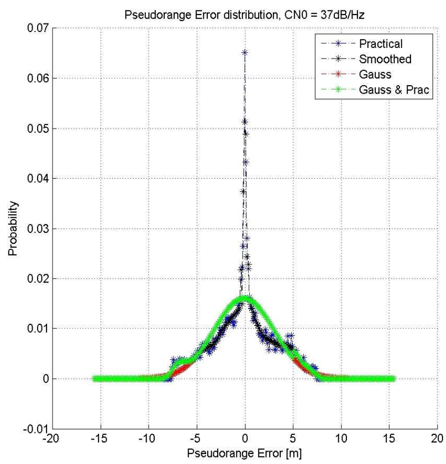

Figure 3. Gaussian error distribution corrected with empirical long tail (Note: the distribution function is not normalized).

III. PVT SOLUTION, ACCURACY ESTIMATION AND INTEGRITY

A. WLS Solution

The Gaussian variance model of the pseudorange errors is used to weight the observations in the PVT solution based on the WLS approach. The system equations [2] may be written as

$$x = [H^T W H]^{-1} H^T W y \qquad (1)$$

where x is a 4-dimensional state vector containing the receiver coordinates and the clock offset, y is the observed minus computed vector, H is the matrix of the partial derivatives and W is the weight matrix defined as the inverse of the measurements variance.

It is possible to evaluate the accuracy of the estimated state vector using the variance error model information, under the assumption that all measurements behave accordingly to the assumed model. The pseudorange errors are mapped into the position uncertainty using variance-covariance matrix of the state estimation

$$C = [H^T W H]^{-1} \qquad (2)$$

As accuracy estimation, we may consider the Distance Root Mean Square (*DRMS*) estimation on the horizontal plane:

$$DRMS = \sqrt{\sigma_n^2 + \sigma_e^2} \qquad (3)$$

where σ_n^2 and σ_e^2 are the coefficient of the variance covariance matrix respectively for north and east directions. As stated in Leick [3] this measure contains from 64% to 77% probability, 2*DRMS* contains about 95% to 98% probability. The hypothesis that all pseudorange measurements behave accordingly to Gaussian error model is not always true due to long tails in the error distribution function. The error estimation for high confidence can not be described by variance covariance matrix (eq. 2)..

B. RAIM/FDE algorithms

The variance error model can also be used to execute a validation procedure looking for one or more outliers affecting the observation data. This is done by mean of global tests, checking if the population of observations shows the expected behaviour.

The global test analyzes the quadratic form $\hat{v}^T \Sigma^{-1} \hat{v}$ of the residuals $\hat{v}^T$ of the pseudorange observations from PT solution. The quadratic form is compared with the threshold value $\chi^2_{1-\alpha,n-p}$ of χ^2 distribution, where $1-\alpha$ is the confidence level of the test and *n-p* are the degrees of freedom of the system [2].

IV. PVT ESTIMATION WITH PARTICLE FILTER APPROACH

Weighted Least Square WLS method is optimal under the assumption that the errors on measurements have a Gaussian distribution. In experimental data, we showed that long tails are present in error distribution and normal function is not correct for errors larger than 2σ (i.e. 95%). When Gaussian

Identify applicable sponsor/s here. *(sponsors)* assumption of pseudorange errors is not satisfied, PF approach [4] is better suited than WLS for PT state vector estimation. With PF approach, outliers are not discarded; they are used in PT state estimation with the assumed occurrence probability (Fig. 1). As pseudorange measurements in low dynamics condition do not satisfy epoch by epoch error independence condition, we consider single epoch PF assuming resampling period equal to one epoch [4]. This approach is also known as Monte Carlo method.

A. *Monte Carlo approach*

The $x = \lfloor x_{ecef}\ , y_{ecef}\ , z_{ecef}\ , off\ \rfloor$ is the state vector that should be estimated, where $x_{ecef}, y_{ecef}, z_{ecef}$, represent the position in ECEF coordinated and off is clock bias of receiver.

In Monte Carlo method, as first step, a discrete search space *s* is generated with all the N admitted solutions $x^r = \lfloor x^r_{ecef}, y^r_{ecef}, z^r_{ecef}, off^r \rfloor$. The number of dimensions of the space is equal to the number of unknown variables. In each dimension a finite set of possible solution is selected, given by

- WLS reference value,
- the interval width

The interval width may be assumed proportional to two parameters

1. the accuracy estimation of the state variables available from the variance-covariance matrix (eq. 2,3)
2. quadratic form $\hat{v}^T \Sigma^{-1} \hat{v}$ of the residuals

1. The accuracy estimation takes into account the effects of geometry (Dilution of Precision DOP) and a-priori quality estimation of received signals, based on signal to noise ratio measurement. Better accuracy implies higher resolution and smaller width of the particle space (the accuracy evaluation does not affect the number of particle).

2. The quadratic form of the residual provides an evaluation of errors affecting the measurements, if redundancy is available. With larger residuals, the selected search space should be wider and the number of particle should be increased.

Given the ρ peseudorange measurement set, for each particle x^r of the search space *s*, the pseudorange measurement error $\Delta\rho^r_i$ for satellite i is estimated (the particle is assume to be the true position of receiver). The error of the measurement is estimated according to equation

$$\Delta\rho^r_i = R^r_i - \rho_i - off^r - \Delta T_i - \Delta I_i \tag{4}$$

Where $R^r_{i_i}$ is the true distance from the satellite i to the particle r, off^r represents the clock bias of particle r, ΔT_i is the troposphere delay, ΔI_i is the ionosphere delay.

The vector of measurement errors of particle r is given by

$$y^r = \lfloor \Delta\rho^r_1, \Delta\rho^r_2, ..., \Delta\rho^r_{Nsat} \ \rfloor \tag{5}$$

Where $Nsat$ is the number of satellite in view. For each particle $x^{r_r} = \lfloor x_l, y_k, z_m, off_n \rfloor$ the probability that the particle represents the right solution is estimated, according to expression.

$$p(x^r|\rho) = p_{S\_1}(\Delta\rho^r_1) \cdot p_{S\_2}(\Delta\rho^r_2) \ldots \cdot p_{S\_Nsat}(\Delta\rho^r_{Nsat}) \tag{6}$$

where p_{Si} is the numerical pseudorange error distribution function for measured signal to noise ratio, experimentally evaluated. The final estimation of the unknown variables $\lfloor x_{ecef}, y_{ecef}, z_{ecef}, off \rfloor$ is the expected mean value in the multi-dimension particles space considering the estimated particles probability distribution.

$$\overline{x_{ecef}} = \frac{\sum_l \left(x^l_{ecef} \cdot \sum_m \sum_n \sum_k p\left(x^{l,m,n,k} | \rho\right) \right)}{\sum_l \sum_m \sum_n \sum_k p\left(x^{l,m,n,k} | \rho\right)} \tag{7}$$

...

Where x^l_{ecef} is the *l*-th sample of particle space *s* in x_{ecef} dimension.

B. *Condition for application of particle filter*

Theoretically, the PF approach can provide better results than WLS only in case of not Gaussian error distribution. Statistical pseudorange error distribution differs from Gaussian distribution only in case of huge errors, greater than 2σ (95%). The use of PF should be limited to those conditions.

The presence of huge errors may be detected with an analysis of post fit residuals, in particular executing a statistical test on the residual quadratic form.

$$\hat{v}^T \Sigma^{-1} \hat{v} > \chi^2_{1-\alpha,n-p} \tag{8}$$

The quadratic form behaves like a χ^2 distribution. If expression (8) exceeds a predefined threshold the PF should theoretically performs better than the WLS.

V. EXPERIMENTAL RESULT WITH PARTICLE FILTER

We have considered 1h acquisition period in a light indoor site with 1 sec epochs. As result, we observe that WLS over-performs the PF in most of cases if no fixes selection algorithm is implemented. This means that the Gaussian is normally a good strategy for error modeling, at least in condition of small errors on pseudorange measurements. For PF implementation we have used hybrid numerical Gaussian error model described in session II. Results are summarized in figure 4 and table I, in the tables the error has been normalized by DMRS (3).

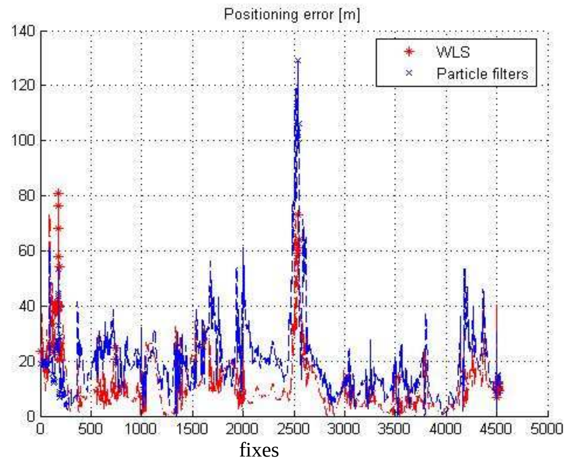

Figure 4. Positioning error using WLS and Particle filter approaches.

TABLE I. NORMALIZED MEAN POSITIONING ERROR OF WLS VERSUS PF

	WLS	Particle Filter
67%	0.3692	0.5944
95%	0.9266	1.2622
Mean	0.3462	0.5656

If the particle PF is applied only on fixes characterized by high values of the residuals quadratic form (8), we can observe improvement in the accuracy of selected fixes as compared with WLS. Higher is the threshold on the fix residuals, more significant is the accuracy improvement with PF. The following figures and tables summarize the results for different confidence values 1-α of χ^2 distribution, α equal to 0.1 and 0.05 have been taken into account.

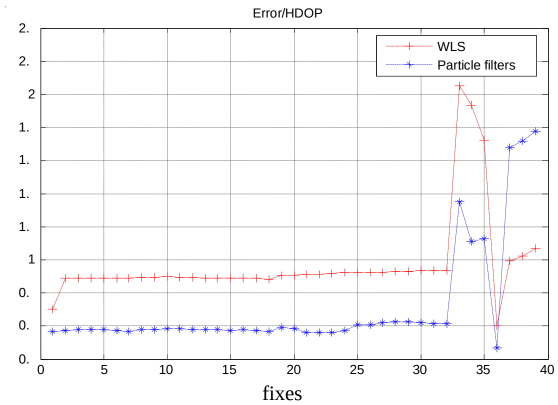

Figure 5. Normalized positioning error for WLS and Particle filter for 40 selected fixes α=0.1

TABLE II. NORMALIZED MEAN POSITIONING ERROR OF PF WITH ALPHA=0.1

	WLS	Particle Filter
67%	0.9242	0.6071
95%	1.8400	1.7024
Mean	0.9781	0.7169

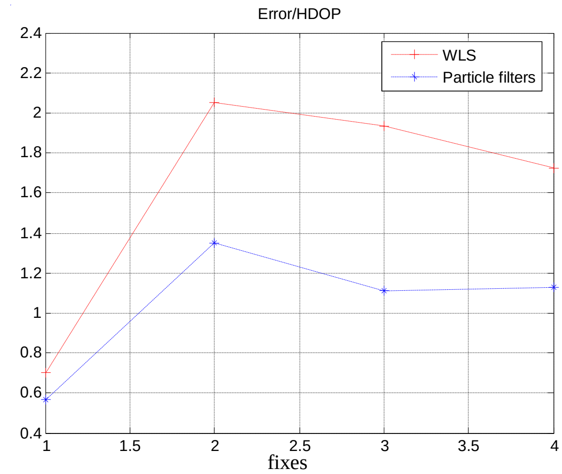

Figure 6. Normalized positioning error for WLS and PF for 4 selected fixes α=0.05

TABLE III. NORMALIZED MEAN POSITIONING ERROR OF PARTICLE FILTERS WITH ALPHA=0.05

	WLS	Particle Filter
67%	1.9543	1.1664
95%	2.0496	1.3527
Mean	1.6030	1.0389

In Table I results, we notice that the assumed error model over estimates the errors affecting measurements acquired in the light-indoor site. This fact might be due to the reason that test measurements have been executed in dynamic conditions whereas the model was estimated in static ones. Further investigations have to be conducted and further improvements may be obtained applying the proposed method.

VI. CONCLUSION

In this work we have shown that in HS GNSS the accuracy of position estimation may be improved with Particle Filter approach, based on a generic multi-environment error distribution. To maximize the effectiveness, Particle Filter should be limited to fixes affected by huge pseudorange measurements errors, when gaussian distribution based on C/N0 values does not represent a reliable model.

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